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# NOMENCLATURE

|            |   |   |
|------------|---|---|
| $\alpha$   | : | Thermal diffusivity of water, $m^2/s$                                   |
| $A$        | : | Cross section area of tube, $m^2$                                       |
| $c_p$      | : | Specific heat of water, $J/Kg.^{\circ}C$                                |
| $b_o$      | : | Width of flattened tubes, $m$   |
| $\delta_I$ | : | Insulation thickness, $m$   |
| $\delta_D$ | : | Tube wall thickness, $m$  |
| $F_r$      | : | Collector heat removal factor   |
| $h_i$      | : | Convective heat transfer coefficient inside of tubes, $W/m^2.^{\circ}C$ |
| $h_w$      | : | Wind heat transfer coefficient, $W/m^2.^{\circ}C$                       |
| $k$        | : | Thermal conductivity of water, $W/m.^{\circ}C$                          |
| $k_I$      | : | Thermal conductivity of insulation, $W/m.^{\circ}C$                     |
| $k_D$      | : | Thermal conductivity of tubes, $W/m.^{\circ}C$                          |
| $L$        | : | Length of tubes, $m$  |
| $\dot{m}$  | : | Mass flow rate of water, $Kg/s$   |
| $n_t$      | : | Number of tubes   |
| $Q$        | : | Quantity of heat absorbed by collector per period, $J$                  |
| $R_i$      | : | Inside radius of tubes before flattened, $m$                            |
| $Re$       | : | Reynolds number   |
| $Re_o$     | : | Reynolds number at the maximum continuous flow rate                     |

$S$  : The rate of incident radiant heat per unit area,  $\text{w/m}^2$   
 $t$  : Time coordinate, s  
 $t_h$  : Heating time, s  
 $t_p$  : Pumping time, s  
 $t_t$  : Time interval of one period, s  
 $T_a$  : Ambient temperature,  $^{\circ}\text{C}$   
 $T_p$  : Outside tube wall temperature,  $^{\circ}\text{C}$   
 $T_1$  : Water temperature at coordinate  $x$  and time  $t$  in free convection,  $^{\circ}\text{C}$   
 $T_2$  : Water temperature at coordinate  $x$  and time  $t$  in forced convection,  $^{\circ}\text{C}$

$\Delta T_{\text{on}}$  : Difference between the temperature at the outlet of the collector and that at bottom of the storage tank at which the pump starts to work,  $^{\circ}\text{C}$

$\Delta T_{\text{off}}$  : Difference between the temperature at the outlet of the collector and that at the bottom of the storage tank at which the pump is shut off,  $^{\circ}\text{C}$

$U_b$  : Heat loss coefficient from the bottom of collector,  $\text{w/m}^2 \cdot ^{\circ}\text{C}$

$U_t$  : Heat loss coefficient from the top of collector,  $\text{w/m}^2 \cdot ^{\circ}\text{C}$

$U_L$  : Total heat loss coefficient taken at the temperature of water,  $\text{w/m}^2 \cdot ^{\circ}\text{C}$

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$U_T$  : Total heat loss coefficient taken at the temperature of outside tube wall,  $W/m^2 \cdot ^\circ C$

$v$  : Water velocity,  $m/s$

$\dot{V}$  : Volume flow rate of water,  $cm^3/s$

$x$  : Position coordinate along the length of collector,  $m$

$\alpha_1$  :  $v \cdot t_f / L$

$\alpha_2$  :  $a \cdot t_f / L^2$

$\alpha_3$  :  $D_o \cdot t_f \cdot U_L \cdot F / c_p \cdot A$

$\alpha_4$  :  $(S/U_L + T_a - T_{in}) / \Delta T_{off}$

$\eta$  : Thermal efficiency of the collector

$\theta_1$  : Non-dimensional temperature in free convection

$\theta_2$  : Non-dimensional temperature in forced convection

$\xi$  : Non-dimensional position coordinate

$\rho$  : Water density,  $Kg/m^3$

$\tau$  : Non-dimensional time coordinate

MARKS ON ORIGINALLY



## CHAPTER I

## INTRODUCTION

The flat solar collector can be used with natural circulation or with forced circulation.

The solar collector heating system is used with natural circulation, using a storage tank located above the collector, with collector and tank connected by a circulation loop. These systems generally lead to low flow rate through the collector with the fluid undergoing a large temperature rise. Because of the low flow rate and high temperature, the system can get boiling phenomenon. However, these systems are reliable, low operating cost and can be operated in remote area where electricity is not available.

The collector heating system is used with forced circulation using a pump to circulate water. The pump suction is connected to the bottom of the storage tank. A temperature control device actuates the on-off operations of the pump. These systems are high in operating cost but commonly used for the heating of buildings.

The collector which was studied in this report was designed by a team of students at Concordia university [1] and won two first awards in water heating by solar energy and in system

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efficiency (including storage), in the International Student Competition of 33 universities, at New Mexico, 1975. The collector was designed with two glass covers. The tubes of the collector were flattened and painted with black enamel paint. Water was used as circulating fluid.

In 1976, Nabil Nicolas [2] used this solar collector system and performed experiments to find the optimum operating condition. However, the experiments failed to determine the precise quantity of mass flow rate which offers the maximum efficiency.

In this report, the optimum operating condition of the flat solar collector with forced circulation is investigated by a numerical method.

## CHAPTER II

## DESCRIPTION OF THE WATER HEATING SYSTEM

The water heating system consists of one flat collector, one storage tank, one circulating pump, one temperature control device and one flow control valve. The system is equipped with a temperature recorder to record continuously the inlet and outlet temperature of the collector. The solar radiation is simulated by a lamp having a power of 1600 watts and a peak energy at  $\lambda = 1.1 \mu\text{m}$  and installed along the length of the collector and parallel to its surface at a distance of 34.5 cm.

Figure 1 shows the picture of the water heating system using the flat collector. Figure 2 shows the schematic diagram of the experimental apparatus [ 2 ].

The components of the flat collector heating system can be described briefly as follows:

## 1. FLAT COLLECTOR :

The flat collector is 91.5 cm wide and 122 cm long. It has 42 tubes of 1.27 cm inside diameter and flattened to an oval cross section and parallel to the length of the collector. Both ends are welded to two horizontal 2.5 cm inside diameter headers at the top and the bottom of the collector. The tubes are coated with a black enamel paint. The collector is covered with 2 pieces of glass which have an air space of 2.54 cm in between. Glass cover having a thickness

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of 3.2 mm and a transmittance of 87%. The wooden frame at the bottom is well insulated by 4 cm fiber glass. The flat collector is inclined  $43^{\circ}$  to the horizontal.

## 2. WATER STORAGE TANK :

Water to be supplied to the collector is stored in an insulated tank in which a constant head is maintained. Tap water from the city water supply circulates continuously through the tank, entering at the bottom and leaving to drain at the top. This continuous flow inside the tank ensures a constant temperature inside the storage tank. This temperature is equal to the temperature of the tap water. The flow rate of tap water can be adjusted.

## 3. TEMPERATURE CONTROL DEVICE :

A temperature control device controls the ON - OFF operation of the circulating pump. The device bases on the temperature difference  $\Delta T$  between water at the outlet of collector and water at the bottom of storage tank. Each setting has two operating points: a  $\Delta T_{on}$  which controls the ON of the pump, and a  $\Delta T_{off}$  which controls the OFF of the pump. The characteristic of this device is graphically presented by the curve  $\Delta T_{on}$  vs  $\Delta T_{off}$  in Figure 3.

## 4. CIRCULATING PUMP :

Water is pumped through the solar collector by a circulating pump. Water enters at the bottom and leaves at the top of collector. The suction of the pump is connected to the bottom of the storage tank.

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5. FLOW CONTROL VALVE :

A valve at the outlet of the collector controls the water flow rate through the collector. Water during the pumping period is collected in a container and weighed accurately. The system is carefully designed to prevent water from flowing through the collector when the pump is stopped.

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## CHAPTER III

## MATHEMATICAL FORMULATIONS

## III.1. FLATTENED TUBE CROSS SECTION AREA, OVERALL HEAT LOSS

## COEFFICIENT, ACTUAL HEAT ABSORBED BY THE COLLECTOR:

We assume that the tubes of the collector are flattened to the form shown in Figure 4. As shown in Figure 4,

$$D_i = d + 2.r \quad (a-1)$$

We assume that the inside wall perimeter keeps the same after flattened:

$$2.\pi.R = 2.d + 2.\pi.r \quad (a-2)$$

We get from equations (a-1) and (a-2) :

$$d = D_i - 2.r$$

$$\text{and } r = (\pi.R - D_i)/(\pi*2)$$

The cross section area of flattened tubes is calculated by the formula:

$$A = 2.d.r + \pi.r^2 \quad (a-3)$$

The heat loss at the top consists of the heat loss by free convection from the glass cover and the heat loss due to the reflective radiation of covers and tube surfaces.

An empirical equation for top loss coefficient is developed by

Klein [3] as follow:

$$U_t = \left[ \frac{N}{\left[ \frac{344}{T_d} \right] \cdot \left[ \frac{T_d - T_a}{N + f} \right]^{0.31}} + \frac{1}{h_w} \right]^{-1} + \frac{\sigma \cdot (T_d^2 + T_a^2)}{\left[ F_d + 0.0425 \cdot N \cdot (1 - F_d) \right]^{-1} + \frac{2 \cdot N + f - 1}{E_g}}^{-1}$$

Where N: number of glass covers

$\sigma$  : Stefan-Boltzmann constant =  $5.67 \cdot 10^{-8} \text{ w/m}^2 \cdot \text{°K}^4$

$h_w$  : wind heat transfer coefficient =  $5.7 + 3.8 V$  [4]

$V$  : wind velocity, m/s

$f = (1 + 0.04 h_w + 5 \cdot 10^{-4} h_w^2) \cdot (1 + 0.058 N)$

$E_g$  : emittance of glass = 0.78

$F_d$  : emittance of tube = 0.9

$T_a$  : ambient temperature, °K

$T_d$  : average outside wall tube temperature, °K

The top loss coefficient can be also read by the graph prepared by J.A. Duffie and W.A. Beckman [5] as shown in Figure 5. We see that its value varies between 2.5 and 5.0  $\text{w/m}^2 \cdot \text{°C}$  for  $T_d$  between 10°C and 110°C.

It is assumed that the temperature at the surface of the bottom of the collector is equal to ambient temperature. The bottom heat loss coefficient  $U_b = k_I / e_I$  where  $k_I$  : thermal conductivity of insulation material,  $e_I$  : insulation thickness. The bottom heat loss coefficient of this collector is equal to:  $U_b = 1.125 \text{ w/m}^2 \cdot \text{°C}$ .

The overall heat loss coefficient taken at the outside tube wall temperature :  $U_T = U_t + U_b$

The actual heat absorbed by the collector per unit area, per unit time is equal to the rate of incident radiant heat minus the total heat loss per unit time, per unit area, written as follow:

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$$q = F_r \left[ S - U_T \cdot (T_p - T_a) \right] \quad (1)$$

where  $S$  is incident radiant heat,  $\text{w/m}^2$ .  $S$  consists of direct solar radiation and diffuse solar radiation. In this case, the energy source is from a lamp. The lamp power is 1600 watts. We assume that the energy coming to the flat collector is 80% of total source energy. Then, the useful source energy is 1280 watts. The surface of collector is equal to  $1.049 \text{ m}^2$ . Then, the incident radiant heat per unit area, per unit time is equal to  $1220 \text{ w/m}^2$ .

$F_r$  is called the collector heat removal factor which defined by N.A. Duffie and W.A. Beckman [5] as:

$$F_r = \frac{\dot{m} \cdot c_D \cdot (T_{wo} - T_{wi})}{N_t \cdot D_o \cdot \Delta x \cdot [S - U_T \cdot (T_p - T_a)]}$$

where  $\dot{m}$  : mass flow rate of water

$c_D$  : specific heat of water

$\Delta x$  : segment along the length of tubes

$N_t$  : number of tubes per collector

$T_{wo}$  : water temperature at the end of segment

$T_{wi}$  : water temperature at the entry of segment

$D_o$  : width of flattened tubes.

The collector heat removal factor depends on the geometry of the collector, the mode of heat transfer between tubes and circulating water, the operating temperature.  $F_r$  will be determined by experimental data in chapter V.

The actual heat absorbed is written in the conduction term

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taken through the tubes:

$$q = k_p \cdot (T_i - T_p) / e_p \quad (2)$$

where  $T_i$  is average inside wall tube temperature

$k_p$  is thermal conductivity of tube

$e_p$  is tube thickness

The actual heat absorbed is written in the convection term taken inside tube:

$$q = h_i \cdot (T_i - T) \quad (3)$$

where  $h_i$  : convective heat transfer coefficient inside tube

$T$  : mean water temperature

From the equations (1), (2), (3), we eliminate  $T_p$  and  $T_i$  and obtain:

$$q = \frac{T_a - T + S/U_T}{\frac{1}{F_r \cdot U_T} + \frac{e_p}{k_p} + \frac{1}{h_i}} \quad (4)$$

If we call  $U_L$  the overall heat loss coefficient calculated with the water temperature, we have:

$$q = F_r \cdot \left[ S - U_L \cdot (T - T_a) \right] \quad (5)$$

From the equations (4), (5), we get:

$$U_L = \frac{S}{T - T_a} + \frac{1 - \frac{S}{U_T \cdot (T - T_a)}}{\frac{1}{U_T} + \frac{F_r \cdot e_p}{k_p} + \frac{F_r}{h_i}} \quad (5-a)$$

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The outlet temperature calculated is not sensitive with the values  $U_L$ . Then, the approximative value of  $U_L$  can be used. We assume that

- in equation (5-a),  $F_r$  is equal to 1. ( $F_r \ll 1$ )
- the mean value of  $U_t$  is used. In Figure 5,  $U_t$  varies between 2.5 and 5.0  $\text{w/m}^2 \cdot ^\circ\text{C}$ . Then,  $U_t$  is equal to 3.75  $\text{w/m}^2 \cdot ^\circ\text{C}$ .
- $U_T = U_t + U_b = 4.88 \text{ w/m}^2 \cdot ^\circ\text{C}$  and  $(T - T_a)$  is equal to  $20^\circ\text{C}$ .

The term  $c_D/k_D = 2 \cdot 10^{-6} \text{ m}^2 \cdot ^\circ\text{C/w}$  may be neglected.

$h_i$  is equal to 300  $\text{w/m}^2 \cdot ^\circ\text{C}$  for free convection and 600  $\text{w/m}^2 \cdot ^\circ\text{C}$  for forced convection.

We obtain  $U_L$  equal to 5.8  $\text{w/m}^2 \cdot ^\circ\text{C}$  for free convection and equal to 5.3  $\text{w/m}^2 \cdot ^\circ\text{C}$  for forced convection.

### III.2. FREE CONVECTION :

#### ENERGY EQUATION:

We observe the control volume shown in Figure 6. The energy equation can be written as follow:

$$\begin{array}{ccccccc} \text{Rate of gain} & = & \text{Rate of energy} & + & \text{Rate of energy} & + & \text{Rate of energy} \\ \text{of energy} & & \text{input by} & & \text{input by} & & \text{input by} \\ & & \text{convection} & & \text{conduction} & & \text{radiation} \end{array}$$

or

$$\begin{aligned} \frac{\partial(\rho C_p \cdot T_1)}{\partial t} \cdot A \cdot \Delta x = & \frac{\partial(\rho \cdot v \cdot C_p \cdot T_1)}{\partial x} \cdot A \cdot \Delta x - \left( -k \cdot \frac{\partial^2 T_1}{\partial x^2} \right) \cdot A \cdot \Delta x \\ & + D_o \cdot \Delta x \cdot F_{r1} \cdot \left[ S - U_L \cdot (T_1 - T_a) \right] \end{aligned} \quad (6)$$

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where  $T_1$  is the temperature of water at position coordinate  $x$  along the length of the collector and at time  $t$ .

We assume that the convection term being much less significant than the conduction term and the radiation term may be neglected during the heating time. Then, the equation (6) becomes:

$$\frac{\partial T_1}{\partial t} = a \cdot \frac{\partial^2 T_1}{\partial x^2} + \frac{D_o \cdot F_r \cdot H_L}{\rho \cdot c_p \cdot A} \cdot \left[ \frac{S}{U_L} - (T_1 - T_a) \right] \quad (6-a)$$

where  $a = k / \rho \cdot c_p$

#### BOUNDARY CONDITIONS:

The temperature at the tube inlet is recorded by the temperature recorder. By experimental data, the temperature at the inlet keeps constant at the storage temperature about 800 seconds, then increases linearly with time. The slope of temperature-time curves at the inlet of the collector is measured. Figure 8 shows the variation of slope of temperature-time curves vs control setting temperature. Because the solutions of the problem are not changed significantly with the values of slope, an average value  $a_o$  of slope can be used.  $a_o$  is equal to  $0.011$   $^{\circ}\text{C/s}$ .

Then, the boundary condition at the inlet of collector can be expressed by following equation:

$$\begin{aligned} \text{at } x = 0, 0 \leq t \leq 800 \text{ seconds, } T_1(0, t) &= T_{in} \\ t > 800 \text{ seconds, } T_1(0, t) &= a_o \cdot (t - 800) + T_{in} \end{aligned} \quad (7)$$

The boundary condition at the outlet of the collector is

simplified as :

$$\text{at the end of tube } x = L, \quad \frac{\partial T_1(L, t)}{\partial x} = 0 \quad (8)$$

INITIAL CONDITION :

At the beginning of each period,  $t = 0$ ,  $T_1(x, 0) = f(x)$ .  
The function  $f(x)$  is unknown. Because the problem was periodical, the temperature distribution along the tube at the end of period must be identical to that at the beginning. Thus,

$$T_1(x, 0) = T_2(x, t_c)$$

Different functions referred to  $f(x)$  are given and the respective temperature distributions  $T_2(x, t_c)$  are obtained. Figures 9a, 9b, 9c show that  $T_2(x, t_c)$  are not sensitive with different functions  $f(x)$ . Then, the iteration method can be applied to find the initial condition. In effect, it requires only two iterations to get the function  $f(x)$  identical to  $T_2(x, t_c)$  as shown in Figure 9d.

### III.3. FORCED CONVECTION

ENERGY EQUATION :

The pump delivers a mass flow rate  $\dot{m}$  equivalent to a mean velocity  $v$  or Reynolds number  $Re$ .

For forced convection, the equation (6) becomes:

$$\frac{\partial T_2}{\partial t} + v \frac{\partial T_2}{\partial x} = a \frac{\partial^2 T_2}{\partial x^2} + \frac{D_o U_L F_{r2}}{\rho \cdot c_p \cdot A} \left[ \frac{S}{U_L} - (T_2 - T_a) \right] \quad (10)$$

# BOUNDARY CONDITIONS ;

The inlet water temperature decreases to the temperature  $T_{in}$  at the bottom of the tank in a very short time .

It is simplified for

$$x = 0 , \quad T_2(0, t) = T_{in} \quad (11)$$

At the outlet of the tubes, it is assumed that the boundary condition of equation (8) is still held:

$$x = L , \quad \frac{\partial T_2(L, t)}{\partial x} = 0 \quad (12)$$

# INITIAL CONDITION :

After a heating time,  $t_h$  , the temperature distribution along the length of collector can be written as:

$$t = t_h , \quad T_2(x, t_h) = T_1(x, t_h) \quad (13)$$

# 111.4: NON-DIMENSIONAL REPRESENTATION :

The non-dimensional variables corresponding to the position coordinate, time coordinate, temperature are defined as :

$$\xi = \frac{x}{L}$$

$$\tau = \frac{t}{t_h}$$

$$\theta_1 = \frac{T_1 - T_{in}}{\Delta T_{off}}$$

and

$$\theta_2 = \frac{T_2 - T_{in}}{\Delta T_{off}}$$

where  $t_t$  : time interval for a period

$T_{in}$  : water temperature at the bottom of the storage tank

$\Delta T_{off}$  : setting temperature at which the pump is shut off.

We replace the non-dimensional variables into system of equations (6a) to (13). We get the system of equations in non-dimensional form as follow :

FOR FREE CONVECTION :

EQUATION :

$$\frac{\partial \theta_1}{\partial \tau} = \alpha_2 \frac{\partial^2 \theta_1}{\partial \xi^2} + \alpha_3 \left[ -\theta_1 + \alpha_4 \right] \quad (6')$$

where  $\alpha_2 = a \cdot t_t / L_o^2$

$\alpha_3 = D_o \cdot t_t \cdot U_L \cdot F_{rl} / \rho \cdot C_p \cdot A$

and  $\alpha_4 = [S / U_L + T_a - T_{in}] / \Delta T_{off}$

BOUNDARY EQUATION :

$$\xi = 0, \quad 0 < \tau < \tau_o, \quad \theta_1(0, \tau) = 0 \quad (7')$$

$$\tau_o \leq \tau, \quad \theta_1(0, \tau) = a'_o \cdot (\tau - \tau_o)$$

where  $a'_o = a_o \cdot t_t / \Delta T_{off}$

$\tau_o = 800 / t_t$

$$\xi = 1, \quad \frac{\partial \theta_1(1, \tau)}{\partial \xi} = 0 \quad (8')$$

INITIAL CONDITION :

$$\tau = 0, \quad \theta_1(\xi, 0) = g(\xi), \text{ determined by iterations. } (9')$$

FOR FORCED CONVECTION :

EQUATION :

$$\frac{\partial \theta_2}{\partial \tau} + \alpha_1 \cdot \frac{\partial \theta_2}{\partial \xi} = \alpha_2 \cdot \frac{\partial^2 \theta_2}{\partial \xi^2} + \alpha_3 \cdot (\alpha_4 - \theta_2) \quad (10')$$

where  $\alpha_1 = v \cdot t_c / L$

BOUNDARY CONDITIONS :

$$\xi = 0, \theta_2(0, \tau) = 0 \quad (11')$$

$$\xi = 1, \frac{\partial \theta_2(1, \tau)}{\partial \xi} = 0 \quad (12')$$

INITIAL CONDITION :

$$\tau = \tau_h, \theta_2(\xi, \tau_h) = \theta_1(\xi, \tau_h) \quad (13')$$

# CHAPTER IV

## NUMERICAL ANALYSIS

The heat transfer process is unsteady and assumed one-dimensional. When the variables  $x$  and  $t$  vary between 0 and  $L$  and between 0 and  $t_e$  respectively, the variables  $\xi$  and  $\tau$  then vary between 0 and 1. We divide 1 of  $\xi$  into  $(L_h - 1)$  non-dimensional segments and 1 of  $\tau$  into  $(K - 1)$  non-dimensional intervals as shown in Figure 7. Then,

one non-dimensional segment is equal to :

$$\Delta \xi = h = 1 / (L_h - 1)$$

one non-dimensional interval is equal to :

$$\Delta \tau = k_t = 1 / (K - 1)$$

the non-dimensional position coordinate is equal to :

$$\xi = (i - 1) \cdot h \quad \text{where } i = 1, 2, 3, \dots, L_h - 1, L_h$$

and the non-dimensional time coordinate is equal to :

$$\tau = (j - 1) \cdot k_t \quad \text{where } j = 1, 2, 3, \dots, K - 1, K$$

The first order partial differential  $\frac{\partial \theta}{\partial \xi}$  is written in backward difference as follow :

$$\frac{\partial \theta}{\partial \xi} = \frac{\theta_{i,j} - \theta_{i-1,j}}{h} \quad (14)$$



# CHAPTER IV

## NUMERICAL ANALYSIS

The heat transfer process is unsteady and assumed one-dimensional. When the variables  $x$  and  $t$  vary between 0 and  $L$  and between 0 and  $t_e$  respectively, the variables  $\xi$  and  $\tau$  then vary between 0 and 1. We divide 1 of  $\xi$  into  $(L_h - 1)$  non-dimensional segments and 1 of  $\tau$  into  $(K - 1)$  non-dimensional intervals as shown in Figure 7. Then,

one non-dimensional segment is equal to :

$$\Delta \xi = h = 1 / (L_h - 1)$$

one non-dimensional interval is equal to :

$$\Delta \tau = k_t = 1 / (K - 1)$$

the non-dimensional position coordinate is equal to :

$$\xi = (i - 1) \cdot h \quad \text{where } i = 1, 2, 3, \dots, L_h - 1, L_h$$

and the non-dimensional time coordinate is equal to :

$$\tau = (j - 1) \cdot k_t \quad \text{where } j = 1, 2, 3, \dots, K - 1, K$$

The first order partial differential  $\frac{\partial \theta}{\partial \xi}$  is written in backward difference as follow :

$$\frac{\partial \theta_{i,j}}{\partial \xi} = \frac{\theta_{i,j} - \theta_{i-1,j}}{h} \quad (14)$$

The first order partial differential  $\frac{\partial \theta}{\partial \tau}$  is written in backward difference as follow :

$$\frac{\partial \theta_{i,j}}{\partial \tau} = \frac{\theta_{i,j} - \theta_{i,j-1}}{k_t} \quad (15)$$

The second order partial differential  $\frac{\partial^2 \theta}{\partial \xi^2}$  is written in finite-difference form as follow :

$$\frac{\partial^2 \theta_{i,j}}{\partial \xi^2} = \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j}}{h^2} \quad (16)$$

We replace the terms (14), (15), (16) into equations (5') to (13') and obtain the difference equations as follows :

FOR FREE CONVECTION :

EQUATION :

$$\begin{aligned} (1 + 2.k_t \cdot \alpha_2 / h^2 + k_t \cdot \alpha_3) \cdot \theta_{i,j} - \theta_{i,j-1} - (\alpha_2 \cdot k_t / h^2) \cdot \\ (\theta_{i-1,j} + \theta_{i+1,j}) &= k_t \cdot \alpha_3 \cdot \alpha_4 \end{aligned} \quad (6'')$$

$\theta_{i,j-1}$  is known and called  $C(i)$ . The system of equations (6'') where  $i = 2, 3, \dots, I_h$  forms a tridiagonal matrix.

BOUNDARY CONDITIONS :

$$\begin{aligned} i = 1, \quad 1 \leq j \leq K_0, \quad \theta_{i,j} &= 0 \\ j \geq K_0, \quad \theta_{i,j} &= A_0 \cdot (j - K_0) \end{aligned} \quad (7'')$$

where  $K_0 = 800 / t_t \cdot k_t$  and  $A_0 = a'_0 \cdot (\tau_h - \tau_0) / (K_1 - K_0)$

$$i = I_h, \quad \theta_{I_h,j} = \theta_{I_h-1,j} \quad (8'')$$

INITIAL CONDITION : It is linearized for 1<sup>st</sup> iteration :

$$j = -1, \quad \theta_{i,1} = h \cdot (i-1) \quad (9'')$$

FOR FORCED CONVECTION :

EQUATION

$$\begin{aligned} (1 + 2 \cdot k_t \cdot \alpha_2 / h^2 + k_t \cdot \alpha_3 + k_t \cdot \alpha_1 / h) \cdot \theta_{2,i,j} - \theta_{2,i,j-1} \\ - (k_t \cdot \alpha_2 / h^2) \cdot \theta_{2,i+1,j} - (k_t \cdot \alpha_2 / h^2 + k_t \cdot \alpha_1 / h) \cdot \theta_{2,i-1,j} = k_t \cdot \alpha_3 \cdot \alpha_4 \end{aligned} \quad (10'')$$

Similarly, the system of equations (10'') forms a tridiagonal matrix.

BOUNDARY CONDITIONS :

$$i = 1, \quad \theta_{2,i,j} = 0 \quad (11'')$$

$$i = L_h, \quad \theta_{2,L_h,j} = \theta_{2,L_h-1,j} \quad (12'')$$

INITIAL CONDITIONS :

$$j = K_1, \quad \theta_{2,i,K_1} = \theta_{1,i,K_1} \quad (13'')$$

The equations (6'') and (10'') are parabolic. Because the term

$\alpha_1$  is too large, the iteration method is not convergent [6].

The simplest way is to solve the problem by the matrix method. The

equations (6'') forms a tridiagonal matrix  $\bar{A}_1$  which is the follow:

$$\bar{A}_1 = \begin{vmatrix} a_1 & -a_3 & & \\ -a_2 & a_1 & -a_3 & \\ & -a_2 & a_1 & -a_3 \\ & & & a_1 & -a_3 \\ & & & -a_2 & a_1 + a_2 \end{vmatrix} \begin{matrix} 2 \\ \\ \\ L_h^{-1} \end{matrix}$$

where  $a_1 = 1 + 2 \cdot k_t \cdot \alpha_2 / h^2 + k_t \cdot \alpha_3$

$a_2 = k_t \cdot \alpha_2 / h^2$

$a_3 = k_t \cdot \alpha_2 / h^2$

Similarly, for forced convection, the tridiagonal matrix is:

$$\bar{A}_2 = \begin{vmatrix} a_1 & -a_3 & & \\ -a_2 & a_1 & -a_3 & \\ & -a_2 & a_1 & -a_3 \\ & & & a_1 & -a_3 \\ & & & -a_2 & a_1 + a_2 \end{vmatrix} \begin{matrix} 2 \\ \\ \\ L_h^{-1} \end{matrix}$$

where  $a_1 = 1 + 2 \cdot k_t \cdot \alpha_2 / h^2 + k_t \cdot \alpha_3 + k_t \cdot \alpha_1 / h$

$a_2 = k_t \cdot \alpha_2 / h^2$

$a_3 = k_t \cdot \alpha_2 / h^2 + k_t \cdot \alpha_1 / h$

The LU decomposition matrix method is applied to solve the problem [7].

The tridiagonal matrix  $\bar{A}_1$  can be decomposed into two matrices: the lower matrix  $\bar{L}_1$  and the upper matrix  $\bar{U}_1$  as follows:

$$\bar{L}_1 = \begin{bmatrix} 1 & & & & \\ m_3 & 1 & & & \\ & m_4 & 1 & & \\ & & & \ddots & \\ & & & m_{L_h-1} & 1 \end{bmatrix} \quad L_h-1$$

$$\text{and } \bar{U}_1 = \begin{bmatrix} u_2 & -a_3 & & & \\ & u_3 & -a_4 & & \\ & & \ddots & \ddots & \\ & & & u_{L_h-2} & -a_3 \\ & & & & u_{L_h-1} \end{bmatrix} \quad L_h-1$$

where  $u_2 = a_1$

$m_i = -a_2/u_{i-1}$  with  $u_{i-1} \neq 0$

and  $u_i = a_1 + m_i \cdot a_3$  for  $i = 3, \dots, L_h-1$

The system of equations (6'') for  $i=2,3,\dots,L_h-1$  can be written in matrix form :

$$\bar{A}_1 \cdot y = b. \quad (17)$$

The vector  $y$  is  $[\theta_{2,j}, \theta_{3,j}, \dots, \theta_{L_h-1,j}]$

The vector  $b$  is  $[b_2, b_3, b_4, \dots, b_{L_h-1}]$

where  $b_2 = \theta_{2,j-1} + (\alpha_2 \cdot k_t / h^2) \cdot \theta_{1,j} + k_t \cdot \alpha_3 \cdot \alpha_4$

$$\text{and } b_i = \theta_{i,j-1} + k_t \cdot \alpha_3 \cdot \alpha_4 \text{ for } i = 3, 4, \dots, L_h - 1$$

We replace matrices  $\overline{L}_1$ ,  $\overline{U}_1$  into equation (17) and obtain :

$$\overline{L}_1 \cdot \overline{U}_1 \cdot y = b \quad (17')$$

$$\text{or } \overline{L}_1 \cdot z = b \quad (18)$$

$$\text{where } \overline{U}_1 \cdot y = z \quad (19)$$

From the equation (18), we obtain the solutions of vector  $z$  as follows:

$$z_2 = 1$$

$$z_i = b_i - m_i \cdot z_{i-1} \text{ for } i = 3, 4, \dots, L_h - 1.$$

Finally, we obtain the solutions of vector  $y$  from the equation (19) as follows:

$$\theta_{L_h-1,j} = z_{L_h-1} / u_{L_h-1}$$

$$\theta_{i,j} = (z_i + a_3 \cdot \theta_{i+1,j}) / u_i \text{ for } i = L_h - 2, \dots, 3, 2$$

Similarly, the same procedure of calculations can be applied to find the solutions of system of equations (10').

The actual energy absorbed is calculated by the trapezoidal rule integration at each step. The formula of the total energy absorbed by the flat solar collector is as follow :

$$Q = m \cdot c_p \cdot \Delta T_{\text{off}} \cdot k_t \cdot \sum_{j=K_1}^{j=K} \frac{1}{2} \cdot (\theta_{L_h, K_1} + 2 \cdot \theta_{L_h, K_1+1} + \dots + 2 \cdot \theta_{L_h, j} + \dots + 2 \cdot \theta_{L_h, K-1} + \theta_{L_h, K})$$

The thermal efficiency of the collector is calculated by the following formula :

$$\eta = \frac{Q/(t_p + t_n)}{S.W.L}$$

The flow chart of the program is shown in Figure 10.

The program is shown in Figure 11.

# CHAPTER V

## RESULTS AND DISCUSSIONS

Four experimental curves of Figures 12,13,14 and 15 obtained from Nicolas' report are used to determine the values of collector heat removal factor  $F_R$ . In effect, the factor  $F_R$  is a parameter of the system of equations (6''), (7''), ... (13''). Then, there is a determined value  $F_R$  which will make the solutions of system of equations (6'') ..., (13'') agree with the experimental results. Because the system of equations is divided into two parts: free convection and forced convection. There will be one value  $F_R$  for each part. To simplify for programming, the value  $F_R$  in free convection must satisfy the following condition:

$$\text{at } t = t_{h \text{ experimental}}, T_1(L, t)_{\text{numerical}} = T_1(L, t)_{\text{experimental}}$$

and the value  $F_R$  in forced convection must satisfy the following condition :

$$\text{at } T_2(L, t)_{\text{numerical}} = T_2(L, t)_{\text{experimental}}, \text{ the actual heat absorbed by the collector per period calculated must be equal to the experimental one: } (Q/\text{period})_{\text{numerical}} = (Q/\text{period})_{\text{experimental}}$$

As shown in Figure 16, the values  $F_R$  are function of the control setting temperature. It can be seen that  $F_R$  increases as the control setting temperature decreases and the value  $F_R$  in forced convection is higher than that in free convection.



We choose  $F_r = 0.41$  for free convection and  $F_r = 0.72$  for forced convection from the curve of Figure 16. These values correspond with  $\Delta T_{on} = 14^\circ\text{C}$ . The experiment on solar collector with the control setting temperature  $\Delta T_{on} = 14^\circ\text{C}$  was carried out. The experimental results of outlet temperature vs time are compared with the calculated values as shown in Figure 17. It is confirmed that the calculated results agreed satisfactorily with the experimental ones.

Figures 12, 13, 14 and 15 also show the variations of the outlet temperature vs time with Reynolds number as parameter.

Figures 18, 19, 20, 21 and 22 show the variations of the thermal efficiency of the collector vs Reynolds number with the control setting temperature  $\Delta T_{on}$  as parameter.

For one fixed value of the control setting temperature, if Reynolds number is very small, the flow is continuous because the temperature at the outlet of the collector becomes steady at the temperature that difference with the temperature at the bottom of the storage tank is higher than  $\Delta T_{off}$ . The Reynolds number corresponding to the maximum continuous flow rate is called  $Re_0$ .

As shown in Figures 19, 20 and 21, the thermal efficiency increases to a maximum and decreases. It can be seen that the thermal efficiency of the collector is maximum at the maximum continuous flow rate. At this point, the efficiency of the

collector is sensitive with the change of Reynolds number.

When the Reynolds number increases to a value higher than  $Re_0$ , the difference between the equilibrium temperature at this flow rate with that at bottom of the storage tank is lower than  $\Delta T_{off}$ . The pump will be shut off and the flow becomes periodical. The theoretical curve of thermal efficiency drops significantly at  $Re = 100$  as can be seen in Figure 21, then increases lightly up to  $Re = 220$ , and decreases lightly. The experimental curve of thermal efficiency is a little different from the theoretical curve. The thermal efficiency drops significantly at  $Re = 100$ , then increases much higher than the theoretical curve at  $Re = 300$ . This disagreement is possibly due to the assumption that  $U_L$  remained constant through the period.

Figure 23 shows that the heat absorbed by the collector per period fluctuates with respect to its constant value  $Q_0$  at very large Reynolds number and that the pumping time gradually decreases as the Reynolds number increases. The efficiency being function of the rate of the absorbed heat also fluctuates with respect to its constant value.

Figures 24, 25 show the variations of the error of the thermal efficiency vs Reynolds number with the control setting temperature  $\Delta T_{on}$  as parameter. It can be seen that the accuracy is reasonable.

Figure 26 shows the variation of  $Re_0$  vs  $\Delta T_{on}$  and the maximum efficiency.  $Re_0$  decreases as the control setting temperature  $\Delta T_{on}$  increases.

Figure 27 shows the maximum efficiency as a function of  $\Delta T_{on}$ .  
The maximum efficiency is nearly inversely proportional to the  
control setting temperature  $\Delta T_{on}$ .

CHAPTER VI

CONCLUSIONS

In conclusion, the finite-difference method was used to solve the problem of the unsteady and periodical heat transfer process. The overall heat loss coefficient was assumed constant. The collector heat removal factor was determined experimentally and used to calculate the actual heat absorbed by the collector. It is seen that the obtained numerical results are quite reasonable.

It may be concluded that :

The thermal efficiency of the collector increases as the control setting temperature  $\Delta T_{on} - \Delta T_{off}$  decreases.

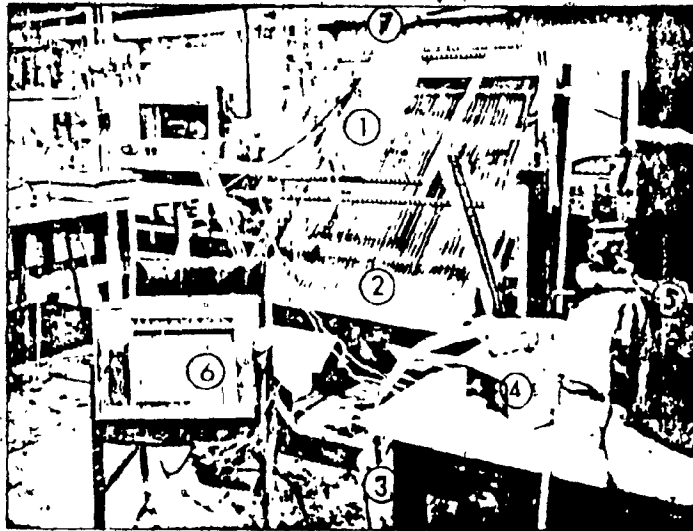
For a fixed control setting temperature, the thermal efficiency of the flat collector is maximum at the maximum continuous flow rate. At this point, the efficiency of the collector is sensitive with the change of the flow rate.

The maximum continuous flow rate increases as the control setting temperature decreases.

This method being general , simple and accurate will be useful to investigate the dynamic performance of the other flat plate solar collectors.

REFERENCES

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- 1 Lamp
- 2 Solar collector
- 3 Pump
- 4 Temperature control device
- 5 Water tank
- 6 Temp. recorder
- 7 Flow control valve

Figure 1 APPARATUS

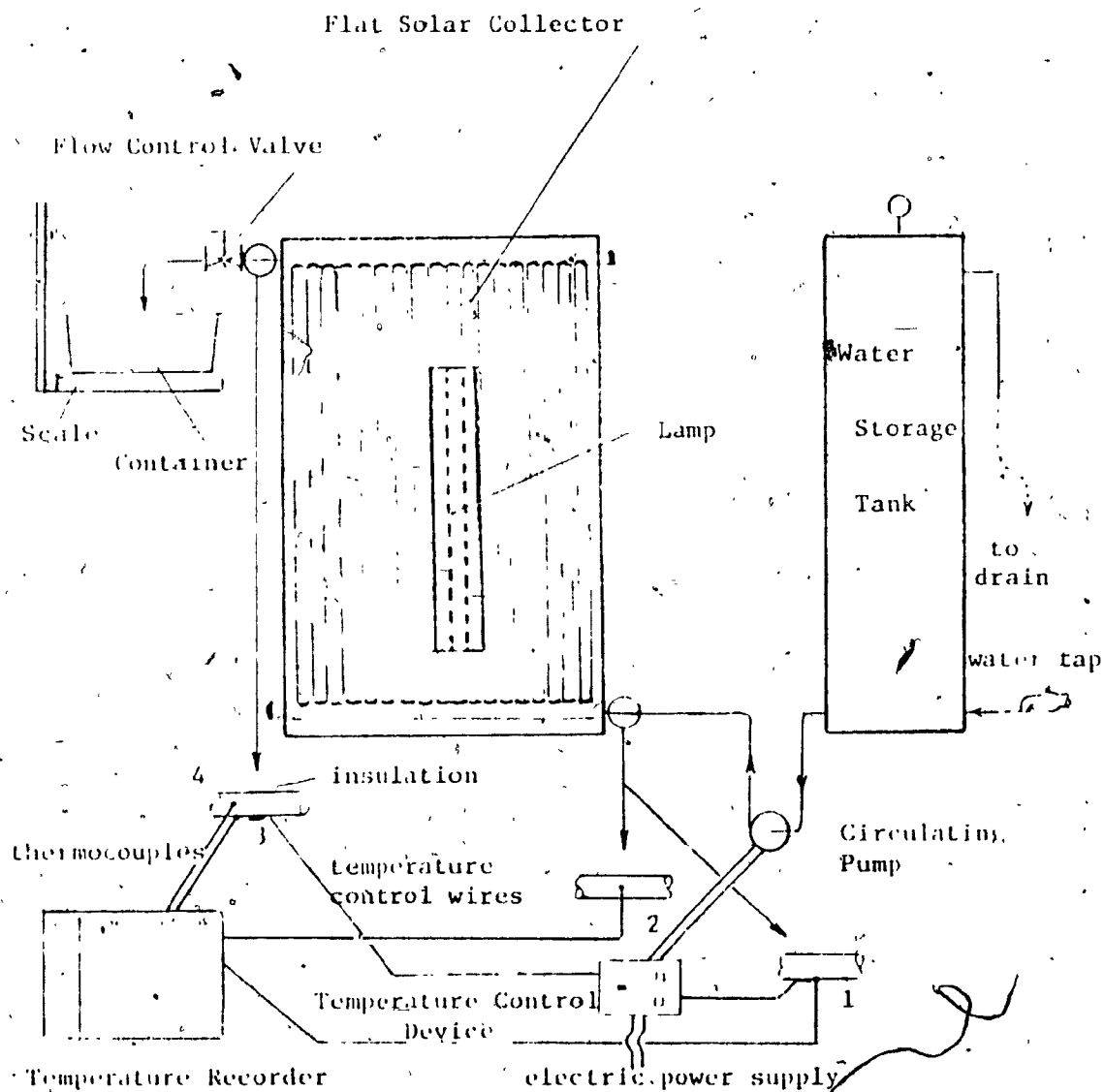


Fig. 2 Schematic diagram of experiment set-up showing solar collector, circulating pump, controls and storage tank.

MARKS ON ORIGINAL

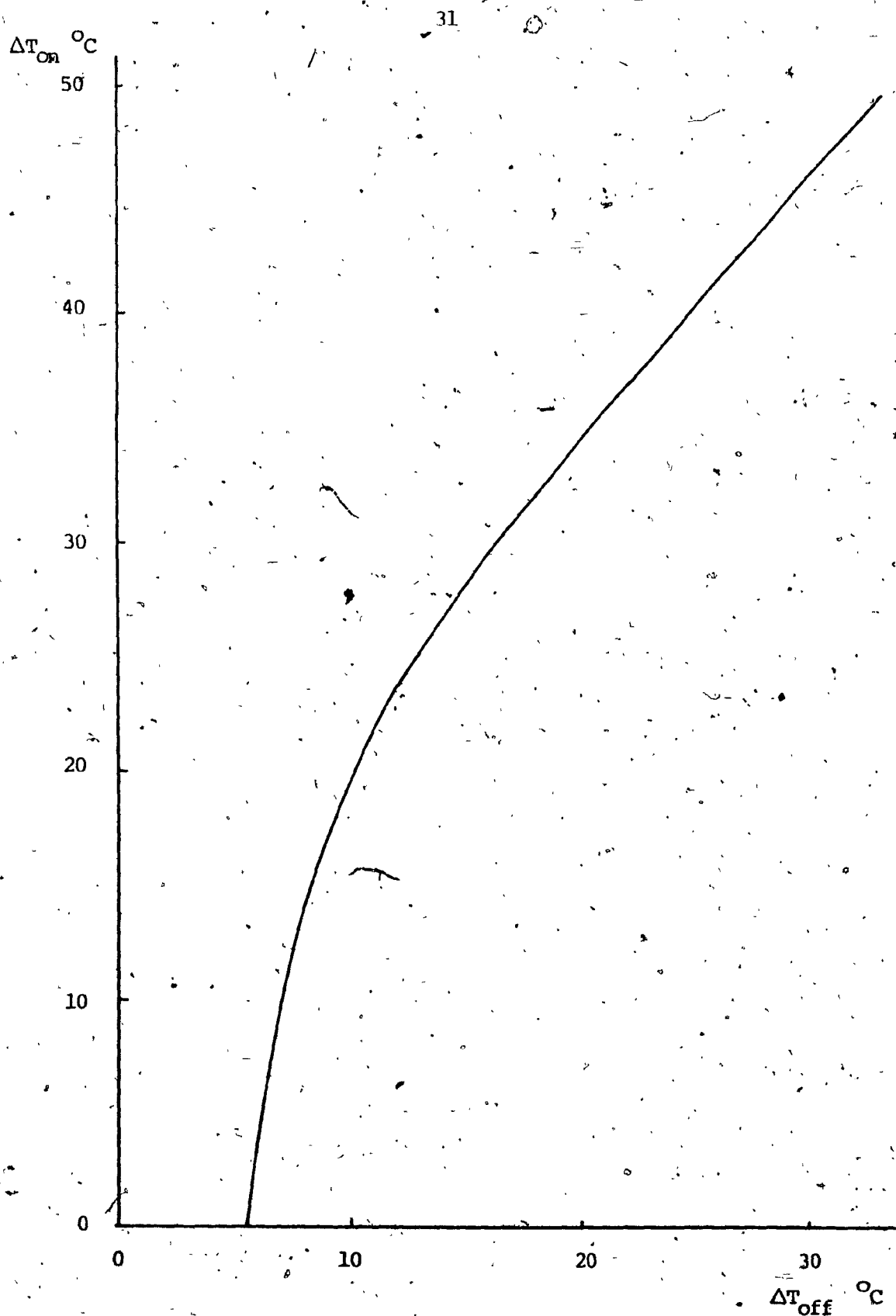


Figure 3. CHARACTERISTICS OF CONTROLLER



MARKS ON ORIGINAL

32

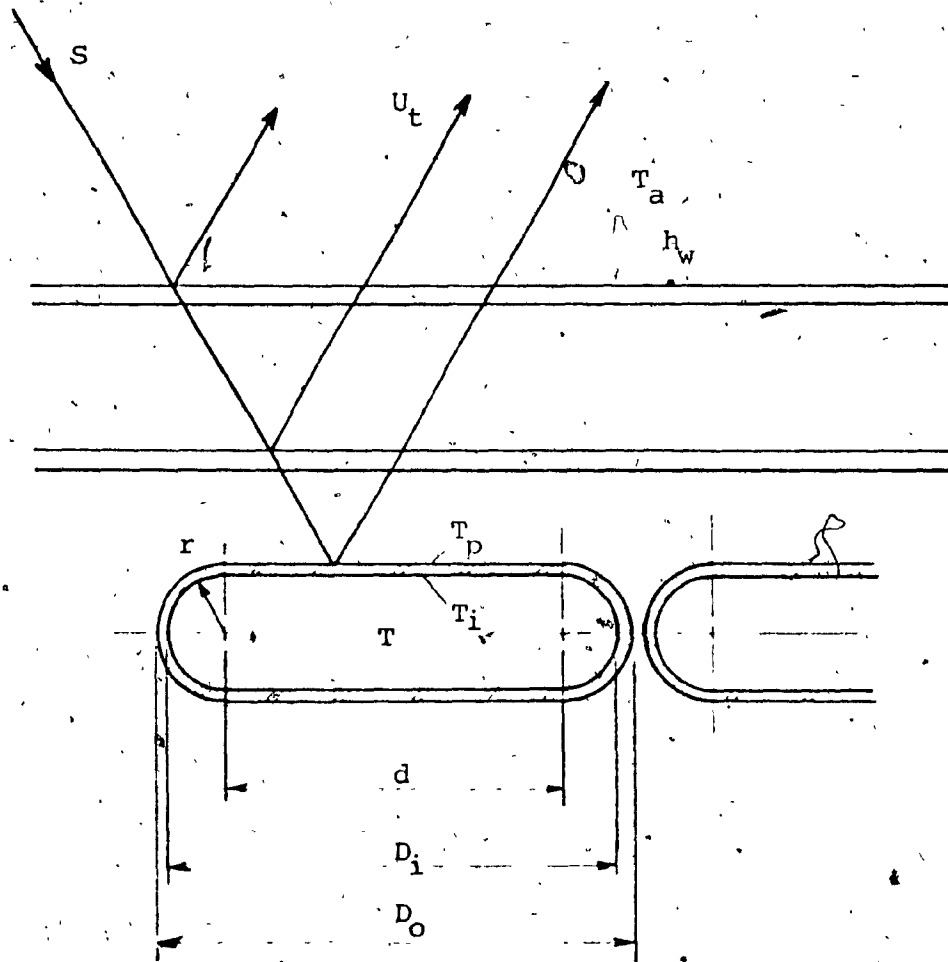


Figure 4. CROSS SECTION OF FLATTENED TUBES OF COLLECTOR

MARKS ON ORIGINAL

33

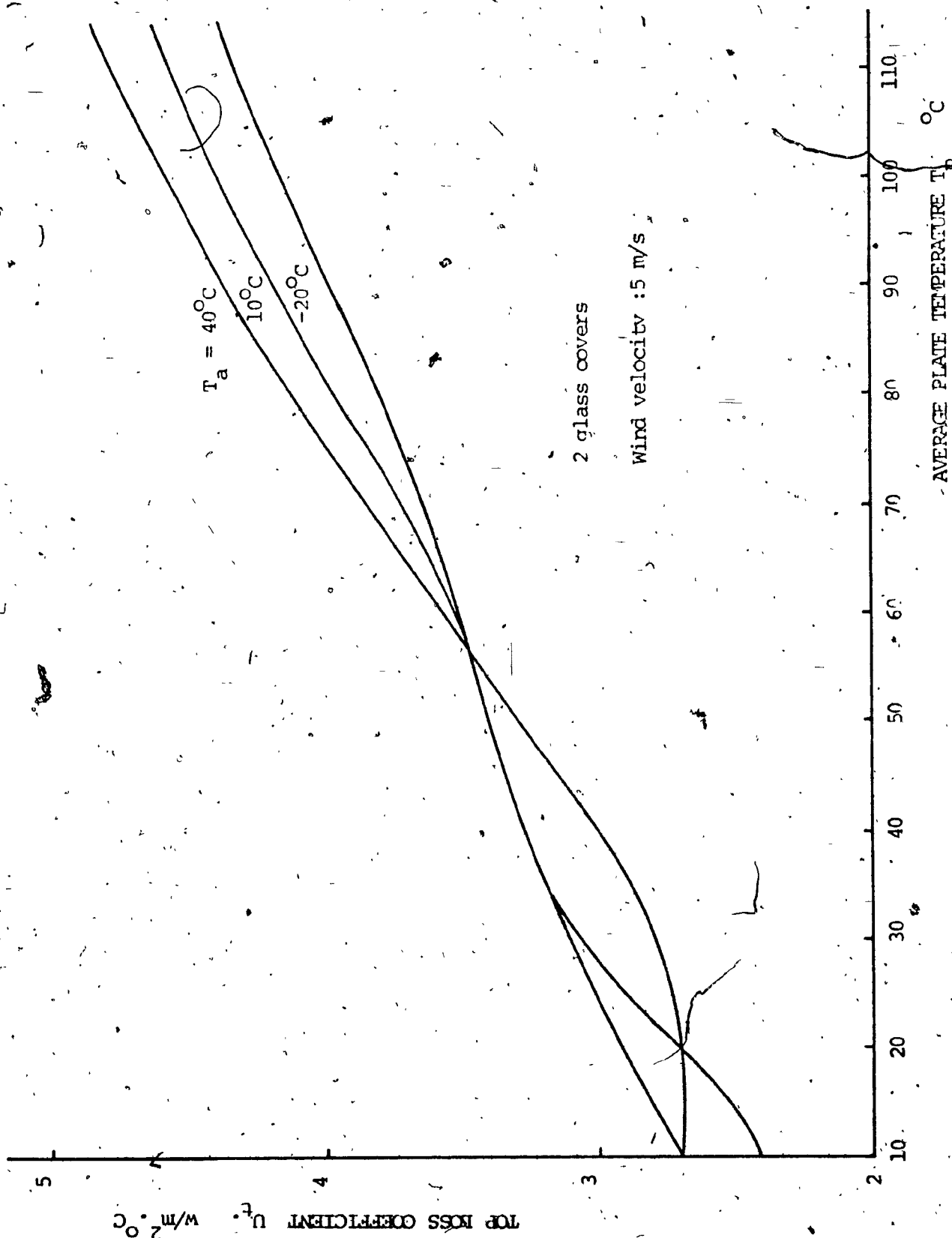
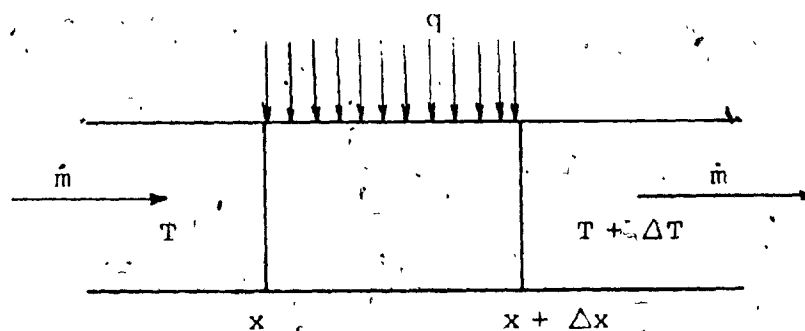


Figure 5



CONTROL VOLUME

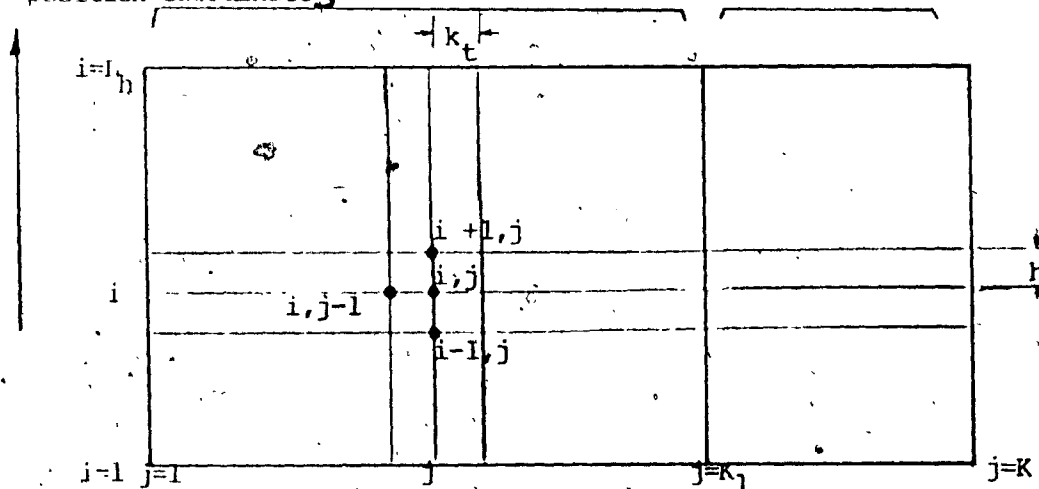
FIGURE 6

Non-dimensional

position coordinate  $\xi$

Free convection

Forced convection



Non-dimensional time coordinate  $\tau$

DIAGRAM OF SEGMENTS IN FINITE - DIFFERENCE

Figure 7

MARKS ON ORIGINAL

35

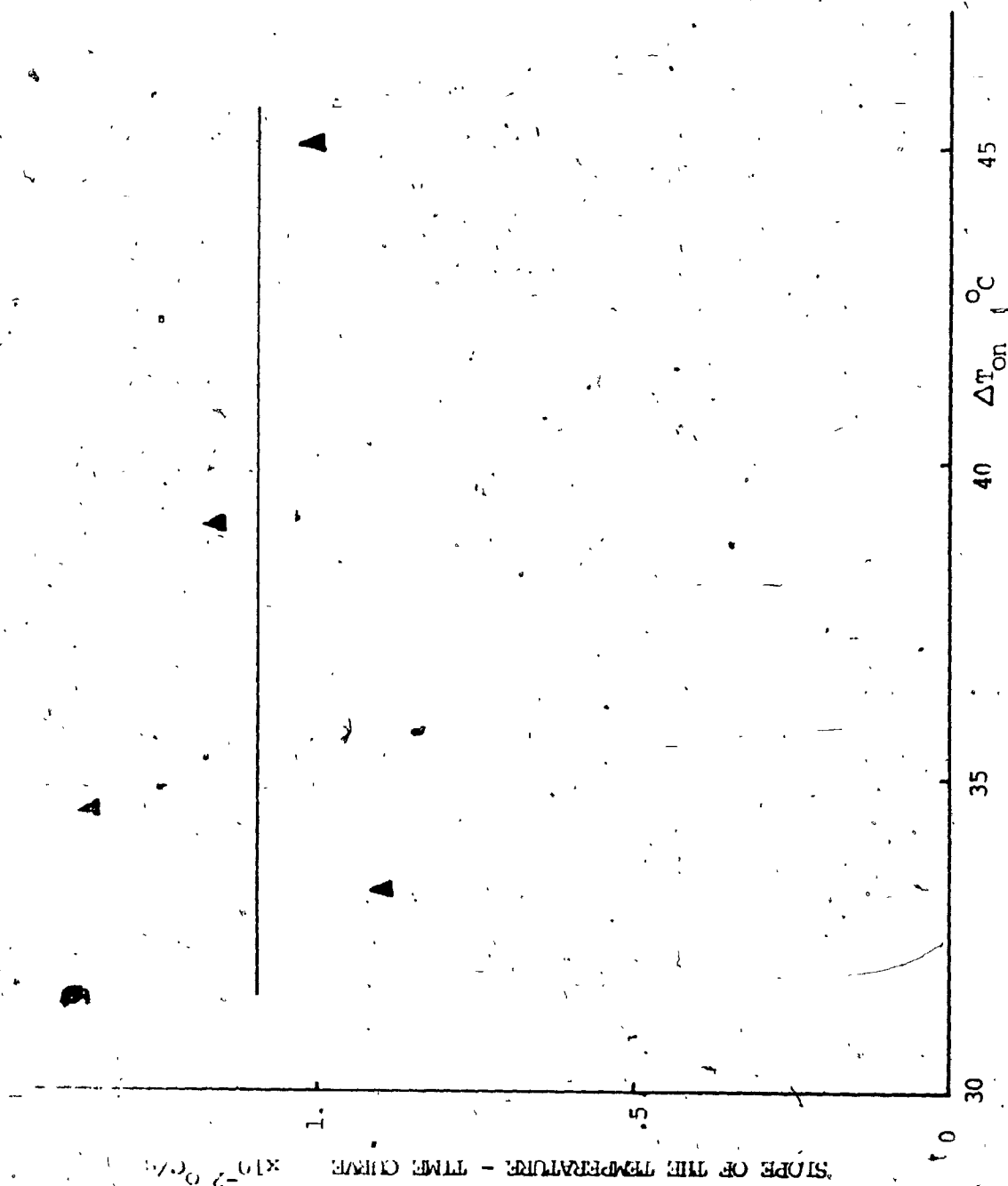


Figure 8. VARIATION OF SLOPE OF THE TEMPERATURE - TIME CURVES VS CONTROL SETTING TEMPERATURE

MARKS ON ORIGINAL

COMPARISON OF TEMPERATURE DISTRIBUTIONS ALONG THE TUBES BETWEEN THE  
BEGINNING AND THE END OF PERIOD

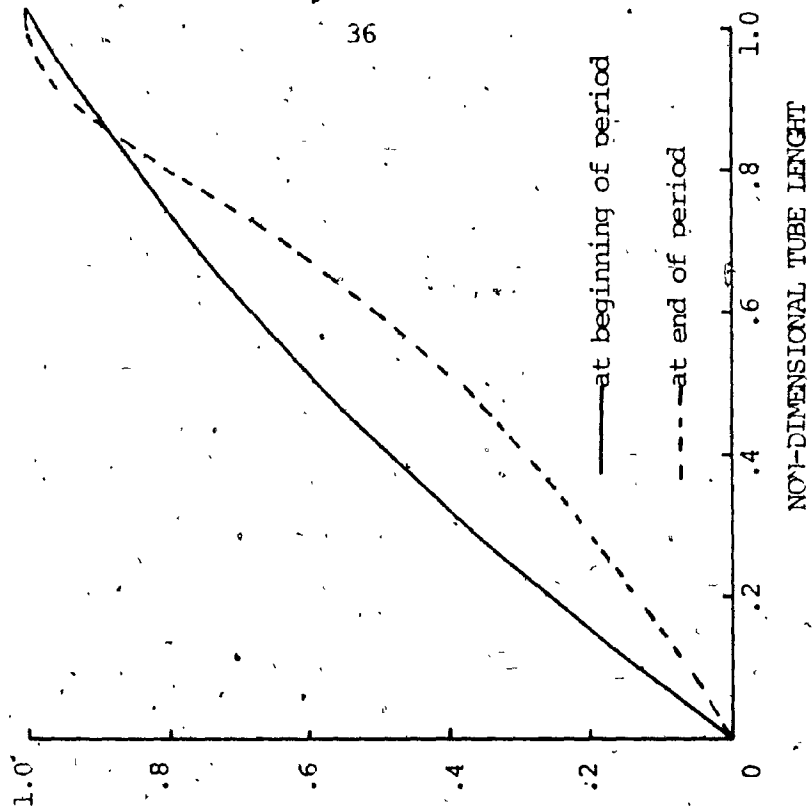


Figure 9a

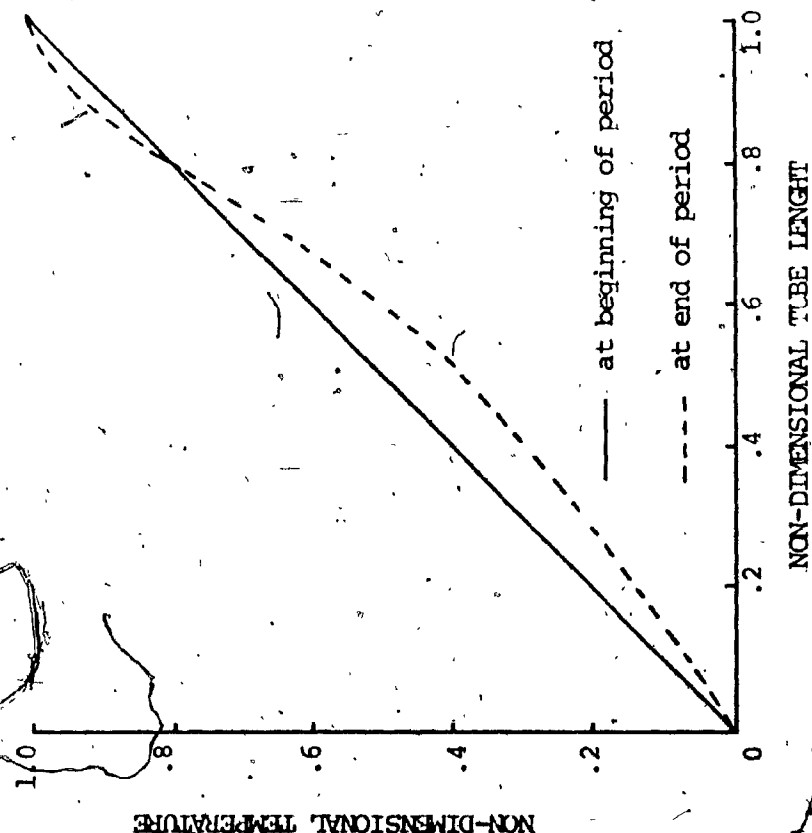


Figure 9b

COMPARISON OF TEMPERATURE DISTRIBUTIONS ALONG THE TUBES BETWEEN  
THE BEGINNING AND THE END OF PERIOD

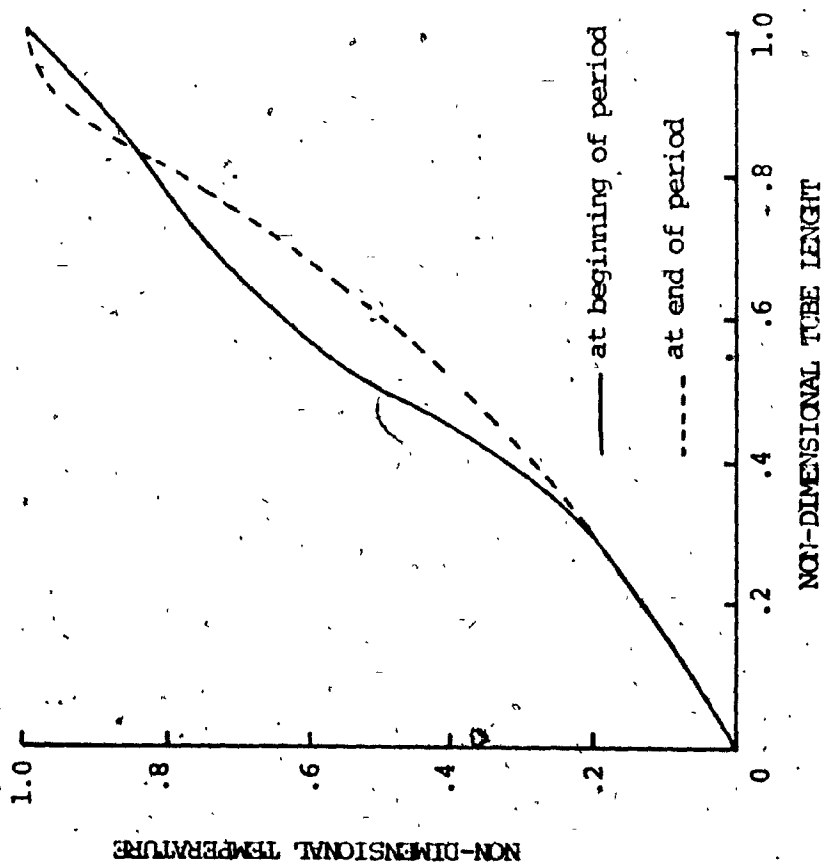


Figure 9c

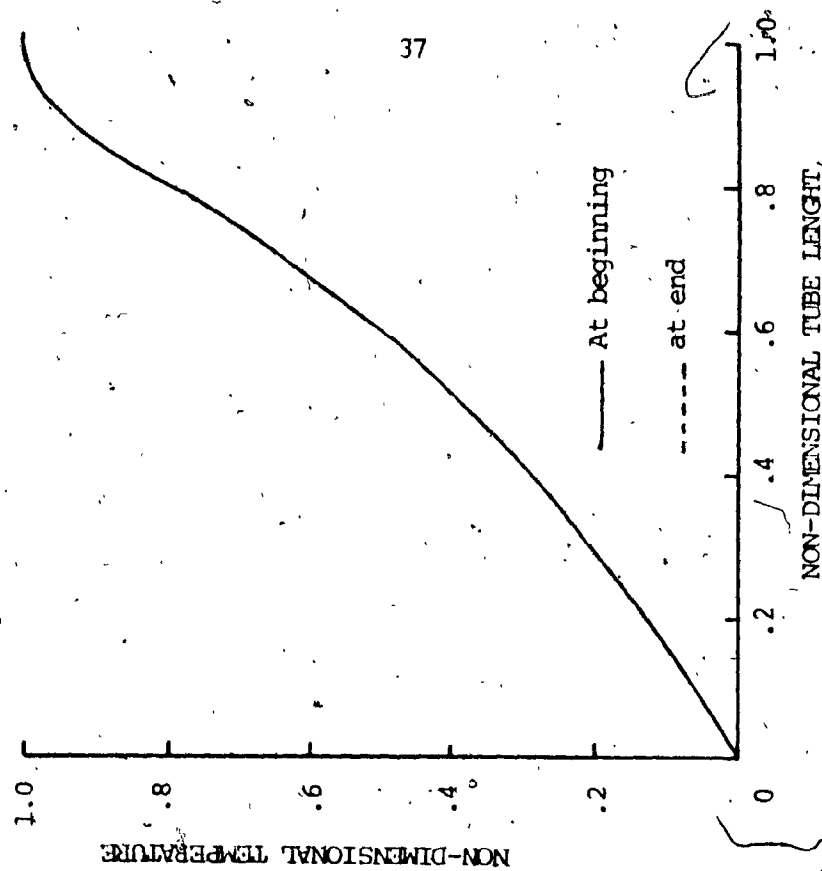
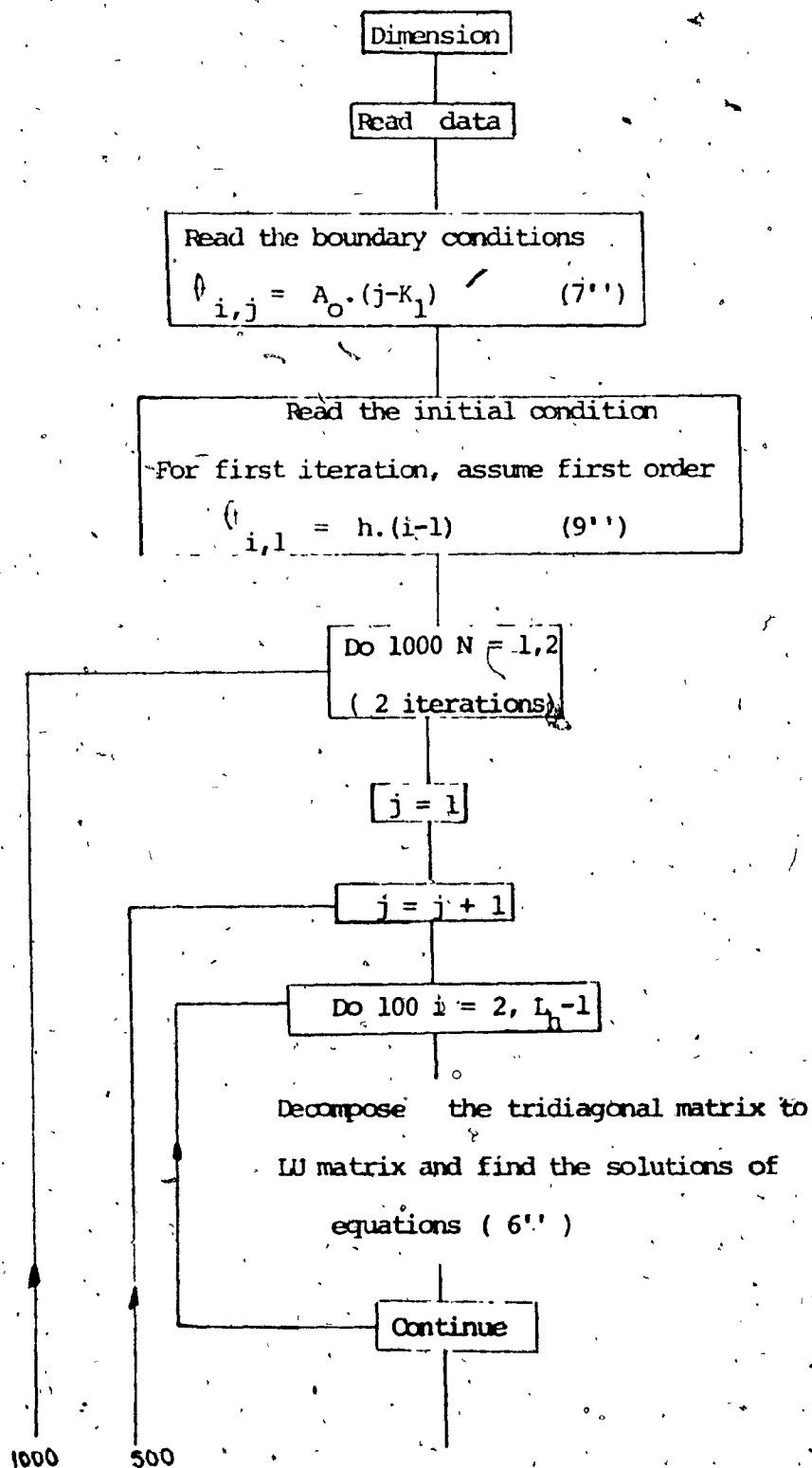
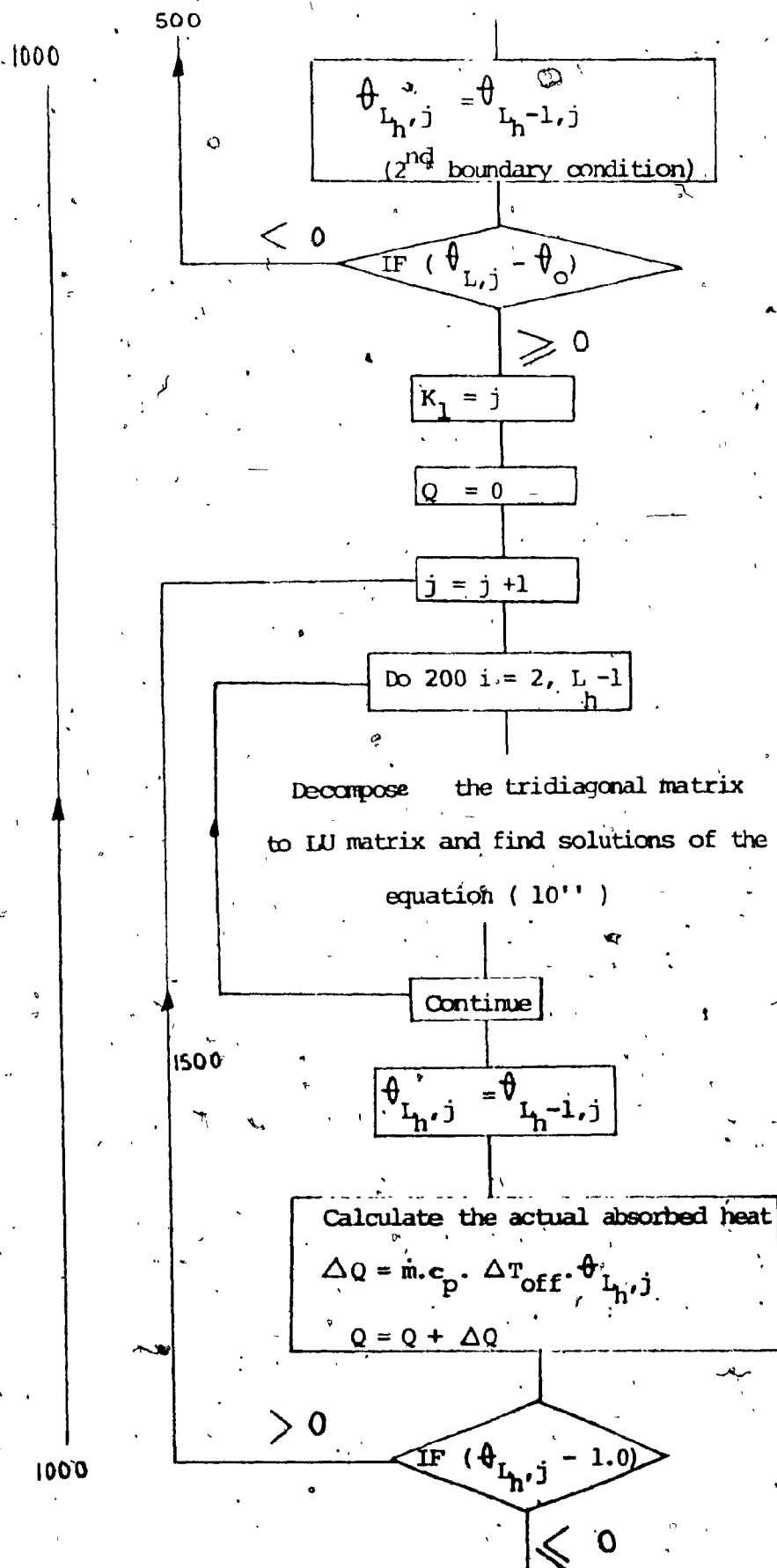


Figure 9d

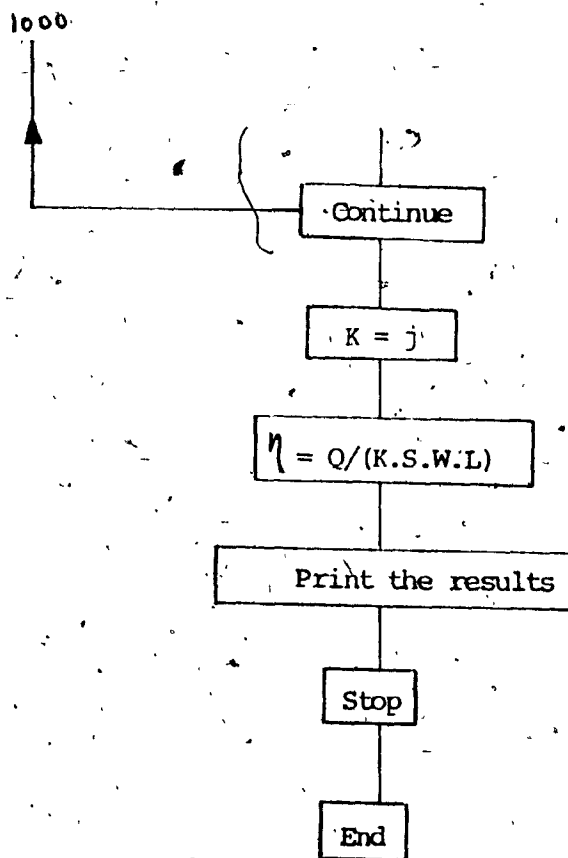
PROGRAM FLOW CHART



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PROGRAM DDTR(INPUT,OUTPUT)

```

C *****
000003 DIMENSION D(400),C(20),R(20),BP(20),U(20),TM(20),TD(20),Y(20),X(20)
      1),TF(20)
C *****
C READ DATA
C
000003 READ 1,L,K,KH,KH2,KH3
000021 READ 2,HSM,GKSM,ALPH1,ALPH2,ALPH3,ALPH4,TFRE,GLEN
C RX1=X1/X2
000045 READ 16,AREA,CP,X2,S,RX1
000063 16 FORMAT(5F13.7)
000063 X1=RX1*X2
000065 X11=X1
000066 K1=K+1
000070 L1=L-1
000072 L2=L-2
000074 L3=L-3
000076 L4=L-4
000100 KH1=KH+1
000102 KH21=KH2-1
000103 KH31=KH3+1
C BOUNDARY CONDITION
000105 DO 210 J=1,KH21
000106 210 D(J)=0.0
000113 DO 220 J=KH2,KH
000115 220 D(J)=D(J-1)+0.0165
000125 DO 230 J=KH1,KH3
000127 230 D(J)=0.0
000134 DO 240 J=KH31,K
000136 240 D(J)=0.0
C INITIAL CONDITION
000143 READ 5,SLOP1,SLOP2,SLOP3
C
C *****
000154 DO 5000 NN=1,8
C *****
000156 PRINT 14,NN
C *****
000163 READ 6,ALPH1,ALPH3,ALPH4,ALPH31,ALPH41
C *****
000201 AMFR=1000.0*AREA*ALPH1+GLEN*42.0/TFRE
C
000206 C(1)=0.0
000211 DO 10 I=2,4
000213 10 C(I)=C(I-1)+SLOP1*HSM
000224 DO 20 I=5,7
000226 20 C(I)=C(I-1)+SLOP2*HSM
000237 DO 30 I=8,11
000241 30 C(I)=C(I-1)+SLOP3*HSM
C
C END DATA
C
000252 R1=GKSM*ALPH2/HSM/HSM
000255 R3=ALPH3*GKSM

```

# MARKS ON ORIGINAL

000257

R4=ALPH3\*ALPH4\*GKSM

42

000261

R51=1.0+2.0\*R1+R3

C  
C  
C  
C  
C

000265 GO TO (515,515,515,515,515,515,515,515),NN  
000302 510 GKSP=GKSM\*1.5  
000304 GO TO 501  
000305 515 GKSP=GKSM  
000307 GO TO 501  
000307 520 GKSP=GKSM/2.0  
000311 GO TO 501  
000312 530 GKSP=GKSM/4.0  
000314 GO TO 501  
000315 540 GKSP=GKSM/6.0  
000317 GO TO 501  
000320 550 GKSP=GKSM/7.0  
000322 GO TO 501  
000323 560 GKSP=GKSM/8.0  
000325 GO TO 501  
000326 570 GKSP=GKSM/12.0  
000330 GO TO 501  
000331 580 GKSP=GKSM/15.0  
000333 GO TO 501  
000334 590 GKSP=GKSM/20.0  
000336 GO TO 501  
000337 501 RP1=GKSP\*ALPH2/HSM/HSM  
000342 RP2=GKSP\*ALPH1/HSM  
000344 RP3=ALPH31\*GKSP  
000346 RP4=RP3\*ALPH41  
000350 R5=1.0+2.0\*RP1+RP3+RP2

C  
C  
C

FOR FREE CONVECTION

000354 A1=R1/R51  
000356 A2=1.0/R51  
000360 A3=R1/R51  
000361 A0=R4/R51  
000363 J=1  
000364 TH=0.0  
000365 450 J=J+1  
000367 Y1=GKSM\*TFRF  
000371 TH=TH+Y1  
000373 IF (J-KH2) 460,470,470  
000376 470 D(J)=D(J-1)+0.0165  
000404 GO TO 460  
000405 460 DO 400 I=2,L1  
000407 400 TD(I)=-A3  
000414 RP(L1)=A1\*D(J)+A2\*C(2)+A0  
000426 DO 500 I=1,L2  
000430 I1=L1-I  
000431 I3=I+1  
000433 RP(I1)=A2\*C(I3+1)+A0  
000442 500 CONTINUE

# MARKS ON ORIGINAL

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```

00444      TF(I)=- (A1+A3)
00451      DO 600 I=2,L2
00453      600 TF(I)=-A1
00460      U(I)=1.0
00463      Y(I)=RP(I)
00470      DO 700 I=1,L2
00471      I3=I+1
00473      TM(I3)=TD(I3)/U(I)
00502      U(I3)=1.0-TM(I3)*TF(I)
00512      Y(I3)=RP(I3)-TM(I3)*Y(I)
00524      700 CONTINUE
00527      DO 800 I=1,L1
00530      I1=L-I
00531      I2=I1+1
00533      I3=I+1
00534      IF (I-1) 301,301,302
00536      302 X(I1)=(Y(I1)-TF(I1)*X(I2))/U(I1)
00554      GO TO 303
00554      301 X(I1)=Y(I1)/U(I1)
00563      GO TO 303
00564      303 C(I3)=X(I1)
00571      800 CONTINUE
00574      C(I)=D(J)
00601      C2=ABS(C(L)-RX1)
00611      IF (C2-0.05) 440,440,450
00614      440 KH=J
00616      GO TO 430

```

C  
C  
C FOR FORCED CONVECTION  
C

```

00616      430 A1=(RP1+RP2)/R5
00622      A2=1.0/R5
00623      A3=RP1/R5
00625      A0=RP4/R5
00626      W=0.0
00627      TP=0.0
00630      SI=-HSM
00632      DO 4100 I=1,L
00633      SI=SI+HSM
00635      FACT=1.0+0.60*SI*(-SI+1.0)
00642      C(I)=C(I)*FACT
00647      4100 CONTINUE
00652      X10=C(L)
00655      420 J=J+1
00657      Y2=GKSP*TFRE
00661      TP=TP+Y2
00663      D(J)=0.0
00666      DO 100 I=2,L2
00670      100 STN(I)=-A1
00675      B(I)=A1*D(J)+A2*C(2)+A0
00707      U(I)=1.0
00712      Y(I)=B(I)
00717      DO 200 I=1,L4
00720      I3=I+1
00722      B(I3)=A2*C(I3+1)+A0

```

# MARKS ON ORIGINAL

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000730 TM(I3)=TD(I3)/U(I)
000737 U(I3)=1.0+TM(I3)*A3
000746 Y(I3)=R(I3)-TM(I3)*Y(I)
000760 200 CONTINUE
000763 R(L2)=A2*C(L1)+A0
000771 TM(L2)=TD(L2)/U(L3)
001000 U(L2)=1.0-A3+TM(L2)*A3
001007 Y(L2)=R(L2)-TM(L2)*Y(L3)
001021 DO 300 I=1,L2
001023 I1=L1-I
001024 I2=I1+1
001026 IF(I1)170,170,180
001030 170 X(I1)=Y(I1)/U(I1)
001037 GO TO 190
001040 180 X(I1)=(A3*X(I2)+Y(I1))/U(I1)
001054 GO TO 190
001054 190 C(I2)=X(I1)
001061 300 CONTINUE
001064 X(L1)=X(L2)
001070 C(L)=X(L1)
001075 C(1)=D(J)
001102 DQ=AMFR*CP*X2*(X10+C(L))*Y2/2.0
001113 W=W+DQ
001115 Z=ABS(X10-C(L))
001125 IF(Z-0.000001)610,610,620
001130 610 W1=AMFR*CP*X2*C(L)
001135 TPER=TH+TP
001137 X1=C(L)*X2
001143 PRINT 23
001146 PRINT 24,X1
001154 GO TO 630
001155 620 X10=C(L)
001160 C1=ABS(C(L)-1.0)
001170 IF(C1-0.05)410,410,420
001173 410 TPER=TH+TP
001175 W1=W/TPER
001177 630 EFFIC=10*.0*W1/S

```

C  
C

```

01202 PRINT 22,X11,X2
01211 PRINT 18,GKSM,GKSP,RX1,X1,X2
01227 PRINT 15,ALPH1,AMFR
01237 PRINT 11,J,KH,TFRE
01251 PRINT 19,TH,TP,TPER
01263 PRINT 17,S,W1,EFFIC
01275 5000 CONTINUE

```

C  
C

```

01277 1 FORMAT(5I4)
01277 2 FORMAT(5F12.10,/,3F14.9)
01277 3 FORMAT(3X,5F13.2)
01277 4 FORMAT(6X,3H1 =,I2,4X,E13.4)
01277 5 FORMAT(3F10.7)
01277 6 FORMAT(5F12.6)
01277 7 FORMAT(10X,6H1E = .I6)
01277 9 FORMAT(2(2X,6E11.4))

```

# MARKS ON ORIGINAL

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001277 11 FORMAT(10X,4HK = ,15.5X,5HKH = ,15.5X,17HTIME REFERENCE = ,F10.3)
001277 13 FORMAT(214)
001277 14 FORMAT(17X,10X,5HNN = ,13,17)
001277 15 FORMAT(10X,8HALPH1 = ,F12.4,5X,14HM.FLOW RATE = ,F12.8)
001277 17 FORMAT(10X,4HS = ,F10.3,4HW = ,F10.3,10X,13HEFFICIENCY = ,F10.3,
18HPER CENT)
001277 18 FORMAT(2X,7HGKSM = ,F10.5,2X,7HGKSP = ,F10.5,2X,6HRX1 = ,F10.5,2X,
15HX1 = ,F10.5,2X,5HX2 = ,F10.5)
001277 19 FORMAT(10X,5HTH = ,F10.2,4HSEC.,5X,5HTP = ,F10.2,4HSEC.,5X,5HTPER=
1,F10.2,4HSEC.)
001277 21 FORMAT(10X,15)
001277 22 FORMAT(10X,9HDT OFF = ,F10.3,10X,8HDT ON = ,F10.3)
001277 23 FORMAT(10X,27HPUMP WORKS CONTINUOUSLY)
001277 24 FORMAT(10X,17HAT TEMPERATURE = ,F8.2)

C
001277 STOP
001301 END

```

NN = 1

PUMP WORKS CONTINUOUSLY

AT TEMPERATURE = 70.20

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 70.19869 X2 =

ALPH1 = 1.0000 M.FLOW RATE = .00229047

K = 280 KH = 51 TIME REFERENCE = 3172.000

TH = 1982.50SEC. TP = 9079.85SEC. TPER = 11062.35SEC.

S = 1280.000W = 673.026

EFFICIENCY = 52.580PER CENT

NN = 2

PUMP WORKS CONTINUOUSLY

AT TEMPERATURE = 54.81

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 54.81106 X2 =

ALPH1 = 1.3097 M.FLOW RATE = .00299983

K = 232 KH = 51 TIME REFERENCE = 3172.000

TH = 1982.50SEC. TP = 7176.65SEC. TPER = 9159.15SEC.

S = 1280.000W = 688.245

EFFICIENCY = 53.769PER CENT

NN = 3

PUMP WORKS CONTINUOUSLY

AT TEMPERATURE = 41.86

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 41.86358 X2 =

ALPH1 = 1.7463 M.FLOW RATE = .00399985

K = 193 KH = 51 TIME REFERENCE = 3172.000

TH = 1982.50SEC. TP = 5630.30SEC. TPER = 7612.80SEC.

S = 1280.000W = 54.750W

EFFICIENCY = 54.750PER CENT

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# MARKS ON ORIGINAL

PUMP WORKS CONTINUOUSLY

AT TEMPERATURE = 28.43

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 28.42736 X2 =  
ALPH1 = 2.6194 M.FLOW RATE = .00599966  
K = 151 KH = 51 TIME REFERENCE = 3172.000  
TH = 1982.50SEC. TP = 3965.00SEC. TPER = 5947.50SEC.  
S = 1280.000W = 713.906  
EFFICIENCY = 55.774PER CENT

NN = 5

PUMP WORKS CONTINUOUSLY

AT TEMPERATURE = 24.50

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 24.49585 X2 =  
ALPH1 = 3.0559 M.FLOW RATE = .00699945  
K = 139 KH = 51 TIME REFERENCE = 3172.000  
TH = 1982.50SEC. TP = 3489.20SEC. TPER = 5471.70SEC.  
S = 1280.000W = 717.686  
EFFICIENCY = 56.069PER CENT

NN = 6

PUMP WORKS CONTINUOUSLY

AT TEMPERATURE = 21.52

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 21.51886 X2 =  
ALPH1 = 3.4925 M.FLOW RATE = .00799947  
K = 130 KH = 51 TIME REFERENCE = 3172.000  
TH = 1982.50SEC. TP = 3132.35SEC. TPER = 5114.85SEC.  
S = 1280.000W = 720.541  
EFFICIENCY = 56.292PER CENT

NN = 7

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 21.51886 X2 =  
ALPH1 = 3.9291 M.FLOW RATE = .00899948  
K = 87 KH = 51 TIME REFERENCE = 3172.000  
TH = 1982.50SEC. TP = 1427.40SEC. TPER = 3409.90SEC.  
S = 1280.000W = 556.229  
EFFICIENCY = 43.455PER CENT

NN = 8

DT OFF = 47.500

DT ON = 19.000

GKSM = .01250 GKSP = .01250 RX1 = 2.50000 X1 = 21.51886 X2 =

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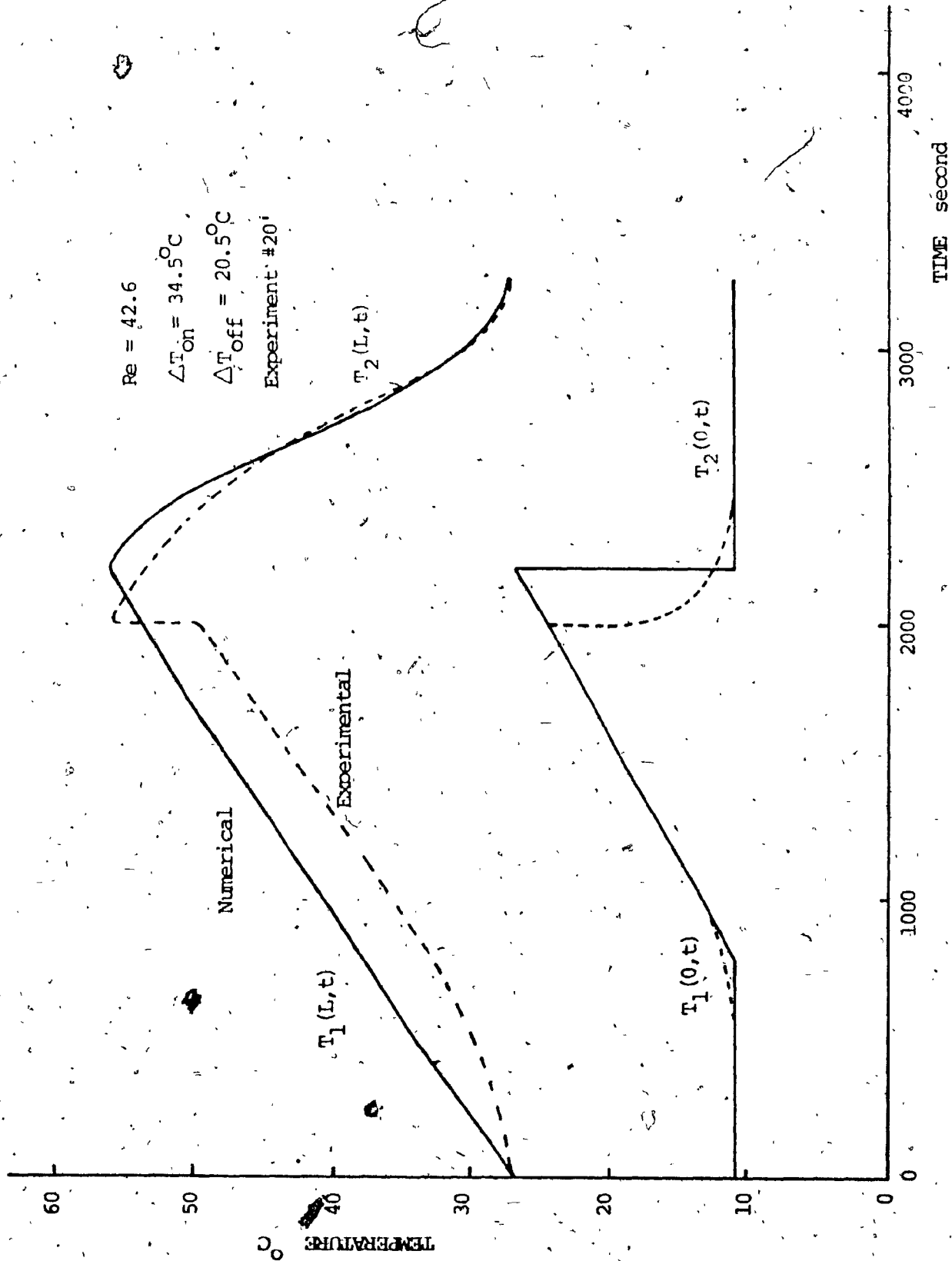


Figure 12. VARIATION OF THE OUTLET WATER TEMPERATURE VS TIME



$Re = 67.1$   
 $\Delta T_{on} = 34.5^\circ C$   
 $\Delta T_{off} = 20.5^\circ C$   
 Experiment # 22

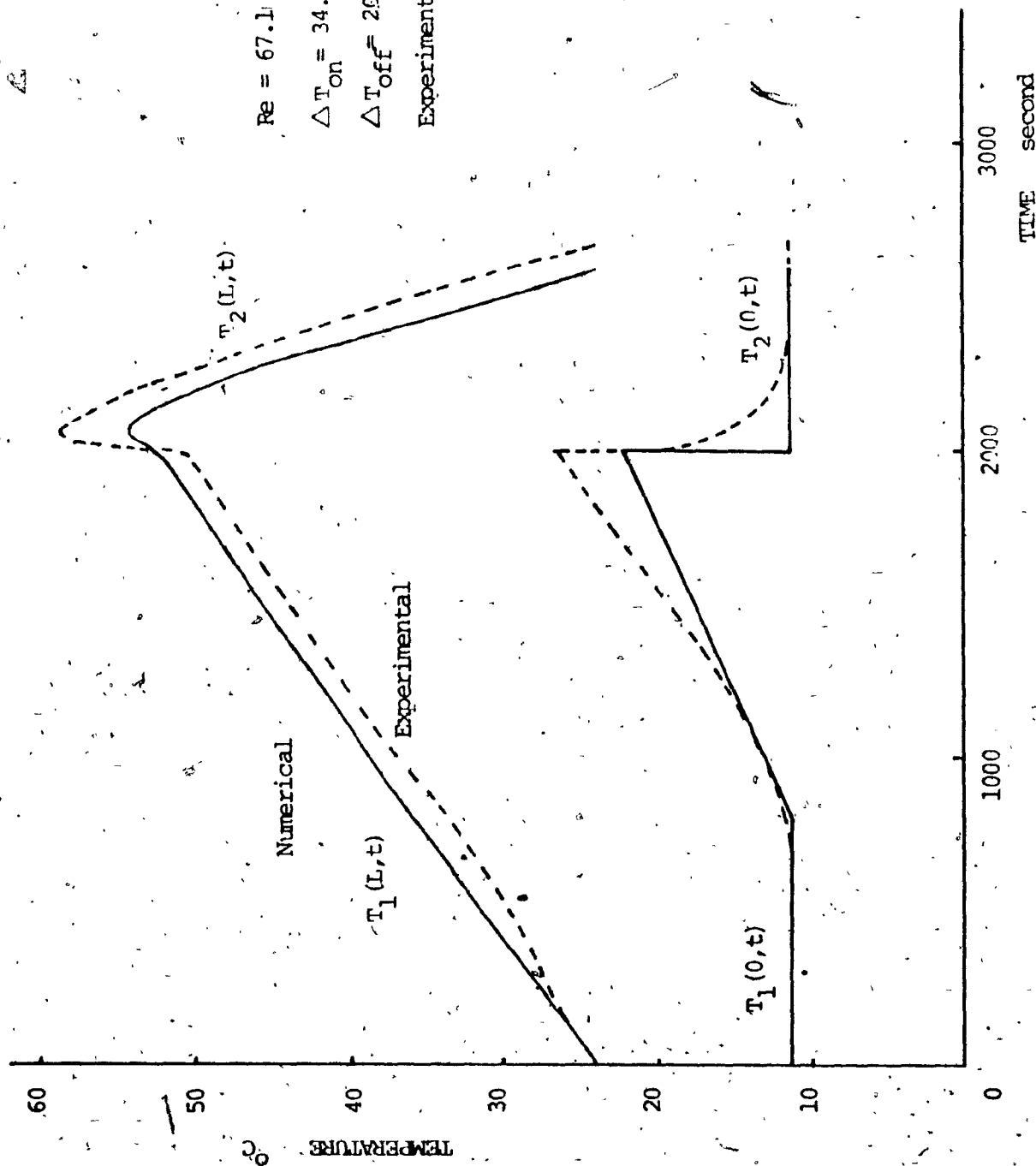


Figure 13. VARIATION OF THE OUTLET WATER TEMPERATURE VS TIME

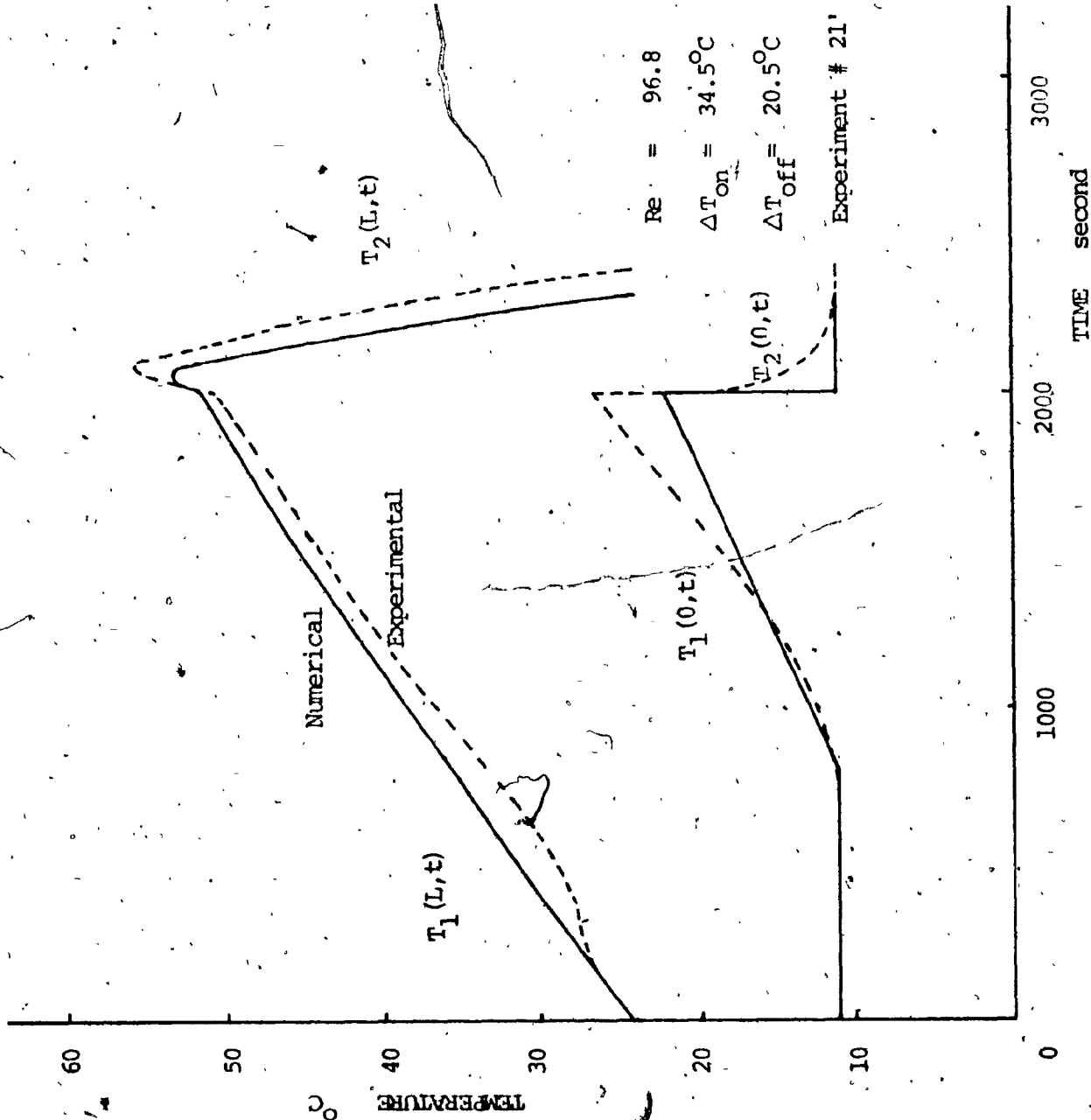


Figure 14. VARIATION OF THE OUTLET WATER TEMPERATURE VS TIME

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$Re = 175.9$   
 $\Delta T_{on} = 34.5^{\circ}C$   
 $\Delta T_{off} = 20.5^{\circ}C$   
 Experiment # 24

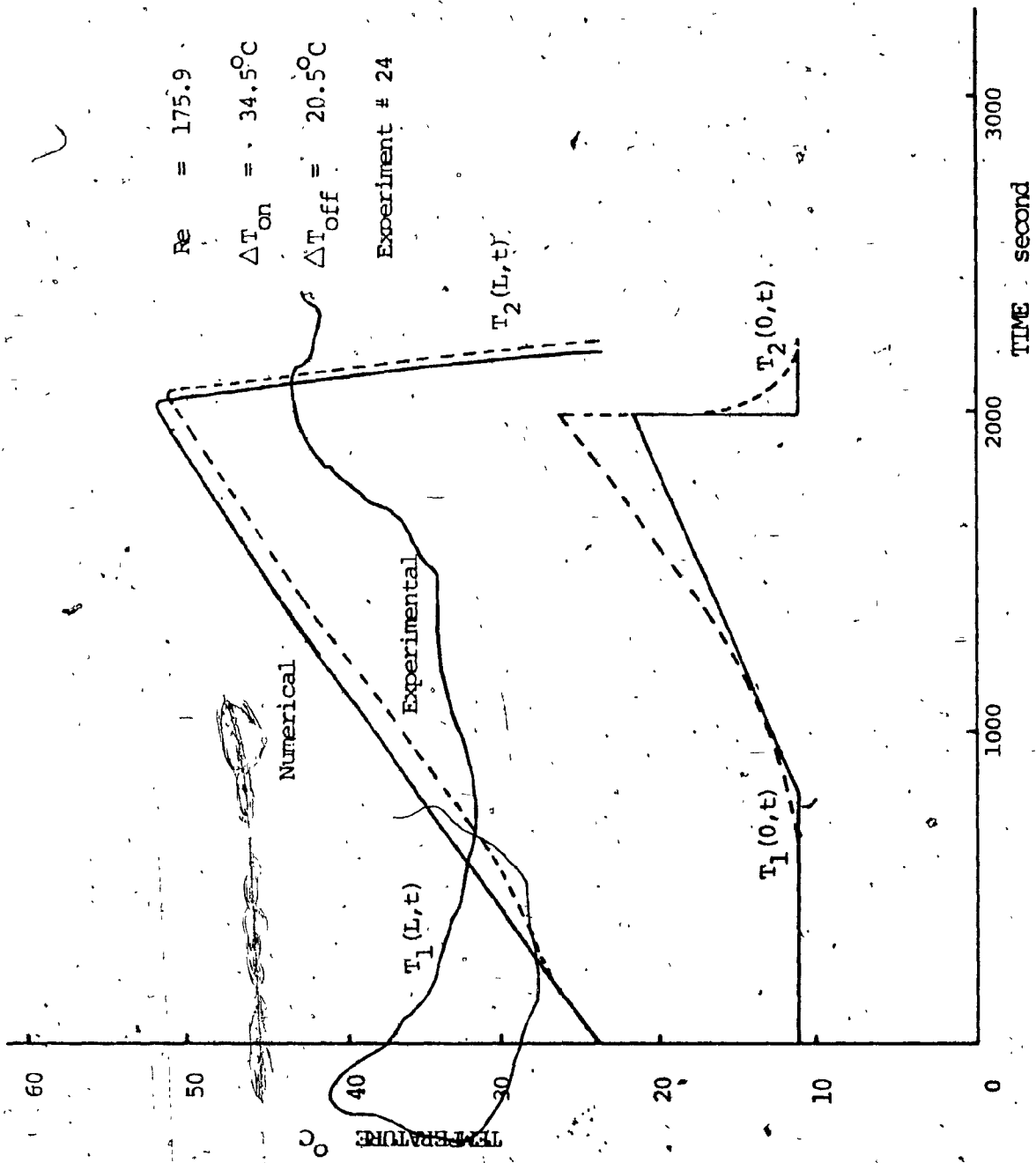


Figure 15. VARIATION OF THE OUTLET WATER TEMPERATURE VS TIME

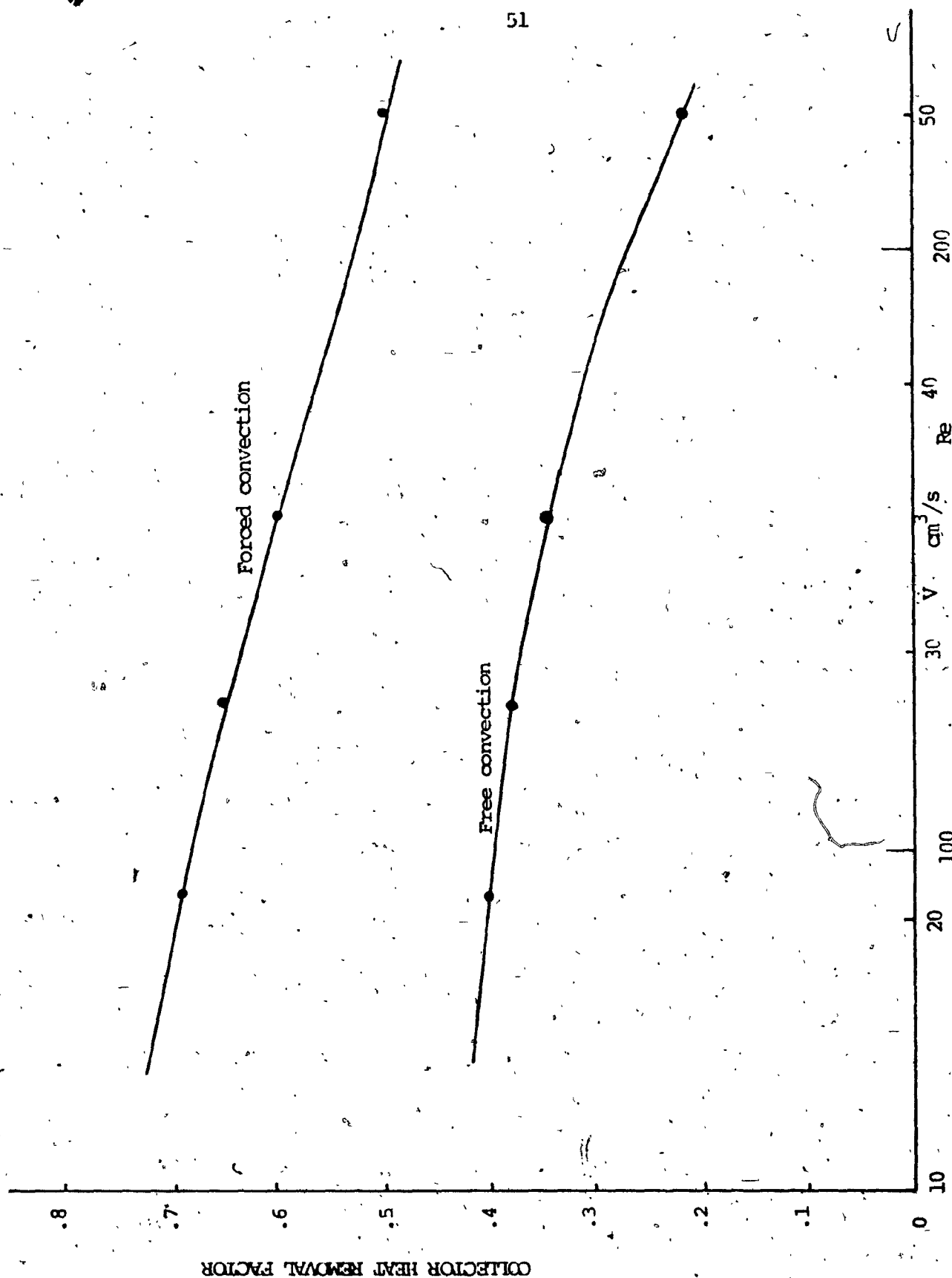


Figure 16. VARIATION OF HEAT REMOVAL FACTOR VS VOLUME FLOW RATE & REYNOLDS NUMBER

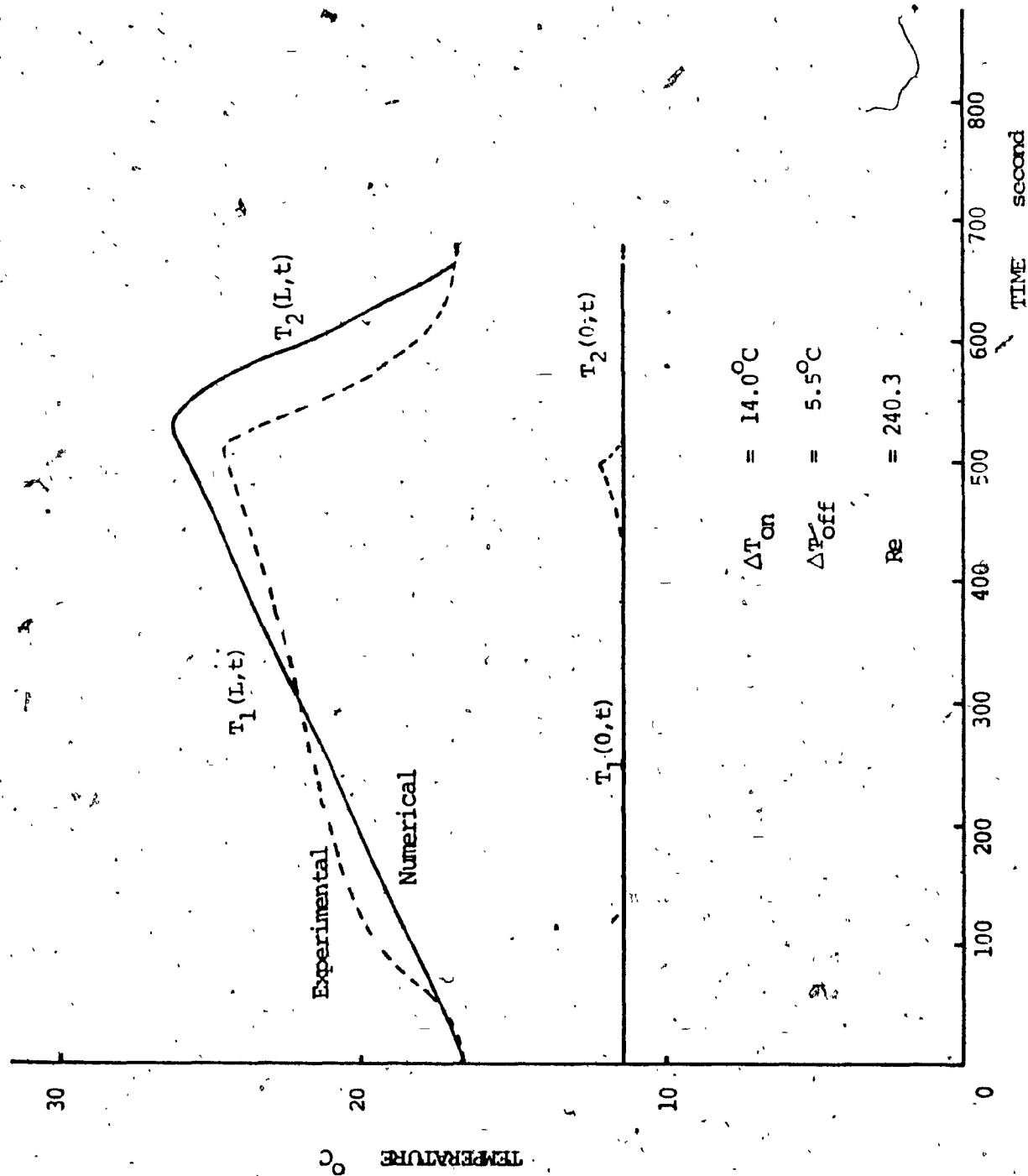


Figure 17. VARIATION OF THE OUTLET WATER TEMPERATURE VS TIME

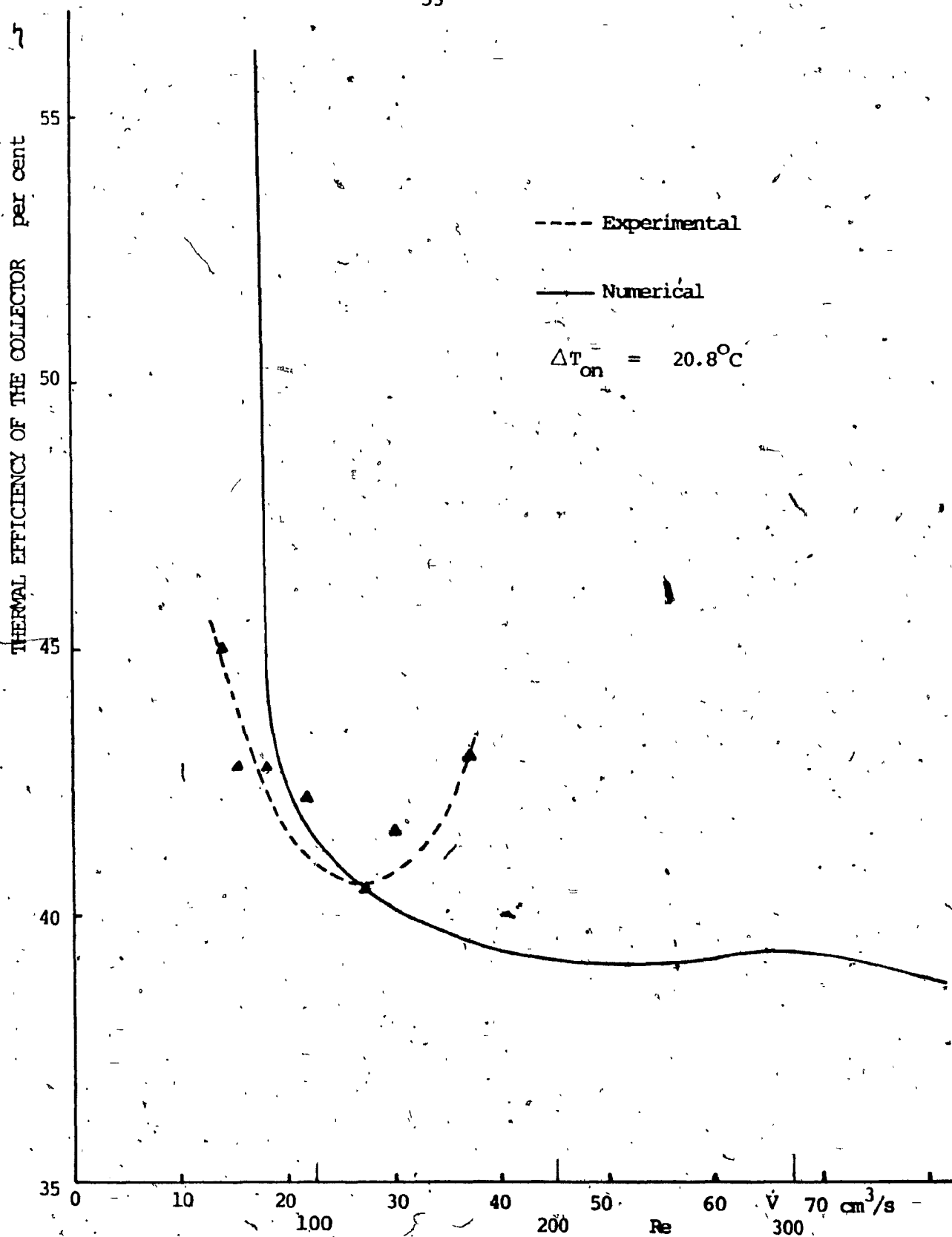


Figure 18. VARIATION OF EFFICIENCY VS VOLUME FLOW RATE & RE NUMBER

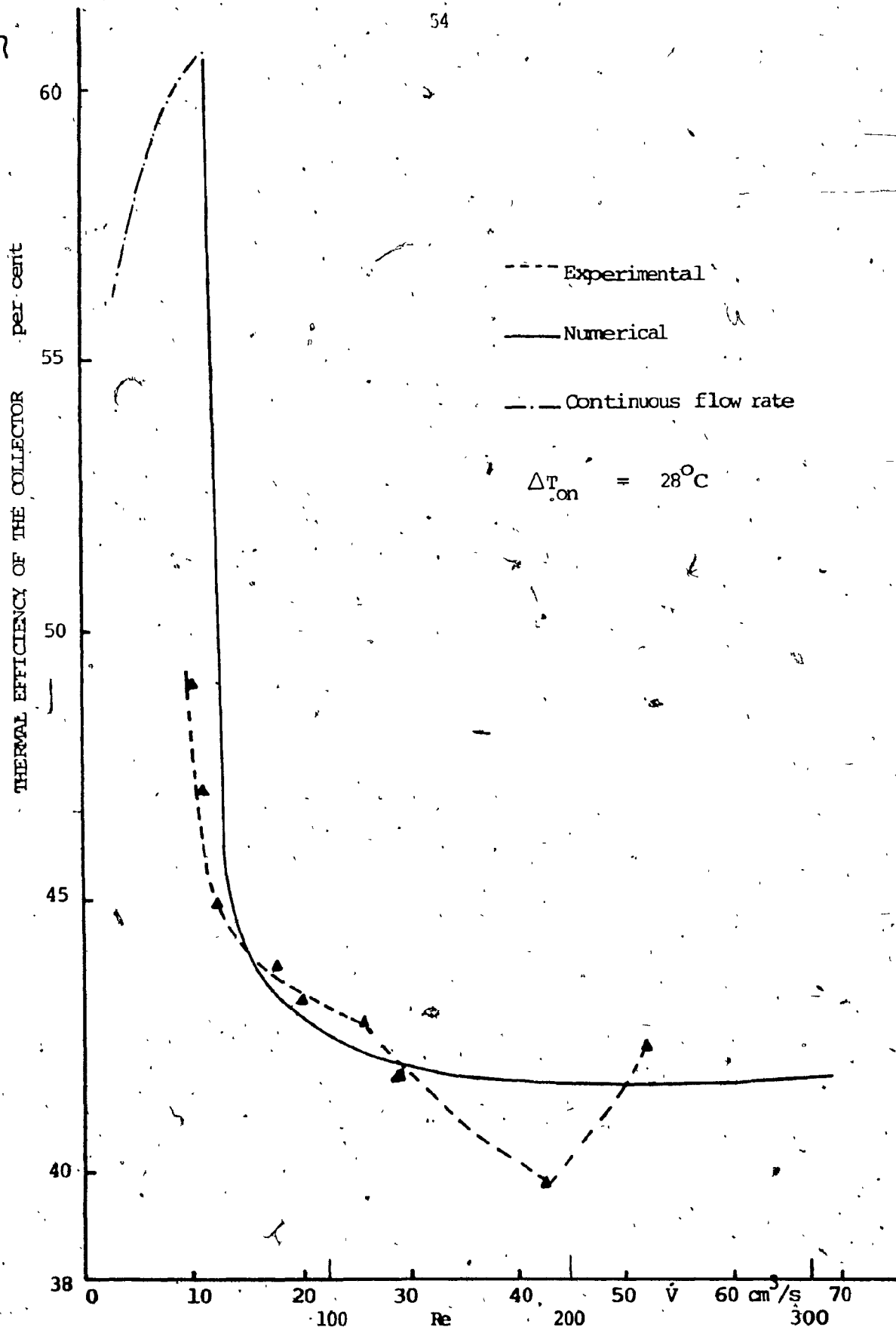


Figure 19. VARIATION OF EFFICIENCY VS VOLUME FLOW RATE & RE NUMBER

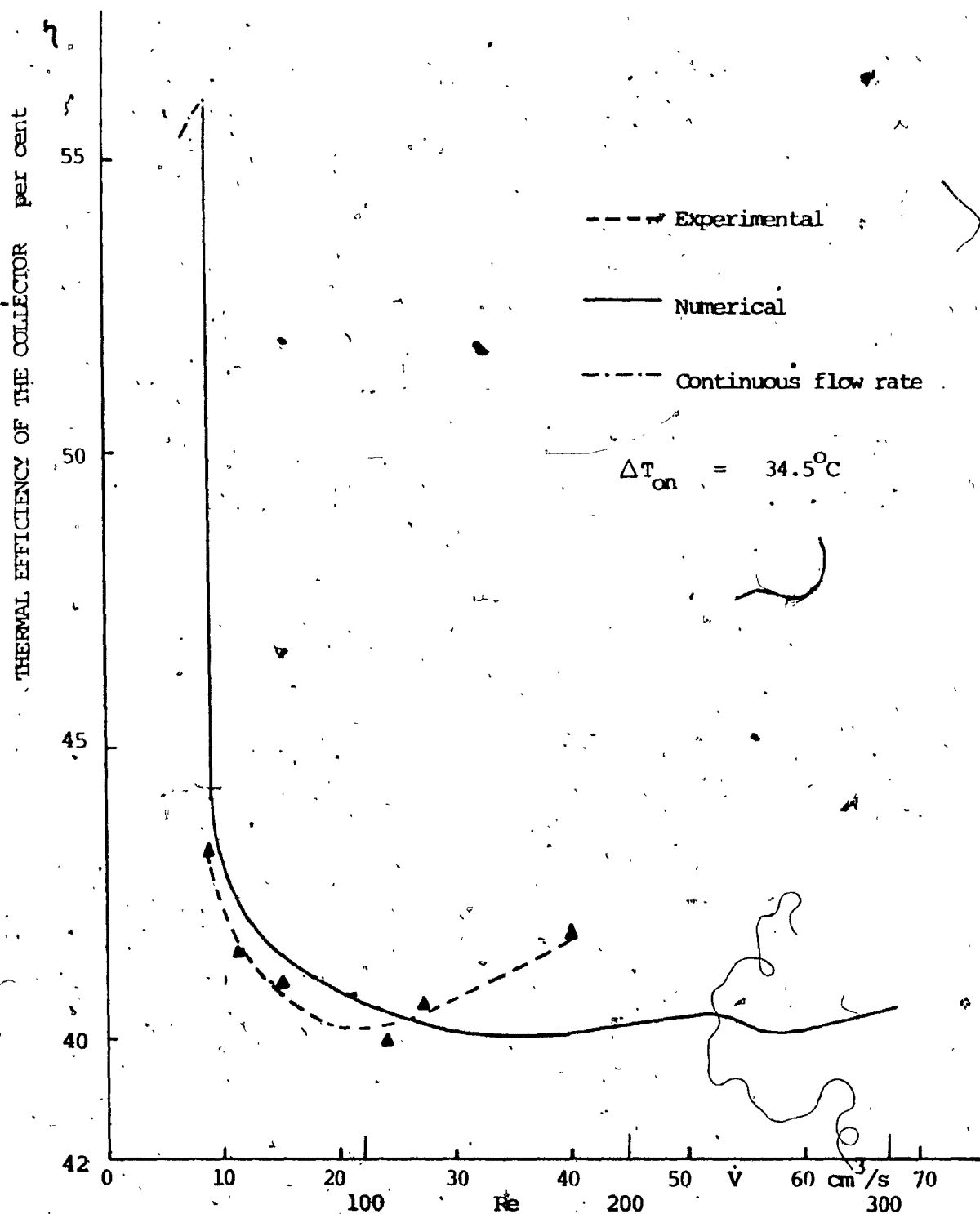


Figure 20. VARIATION OF EFFICIENCY VS VOLUME FLOW RATE & RE NUMBER



MARKS ON ORIGINAL

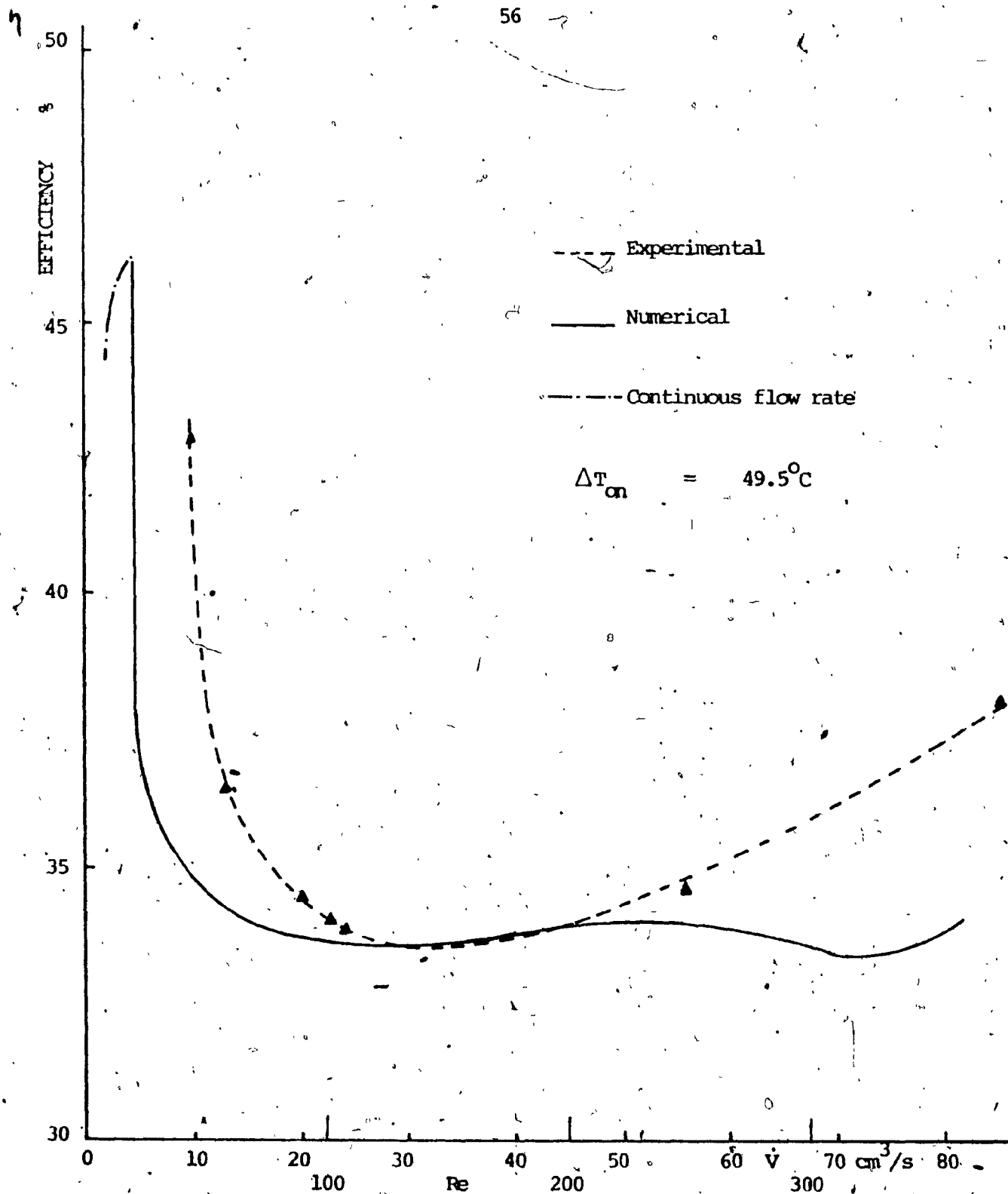


Figure 21. VARIATION OF EFFICIENCY VS VOLUME FLOW RATE & RE NUMBER

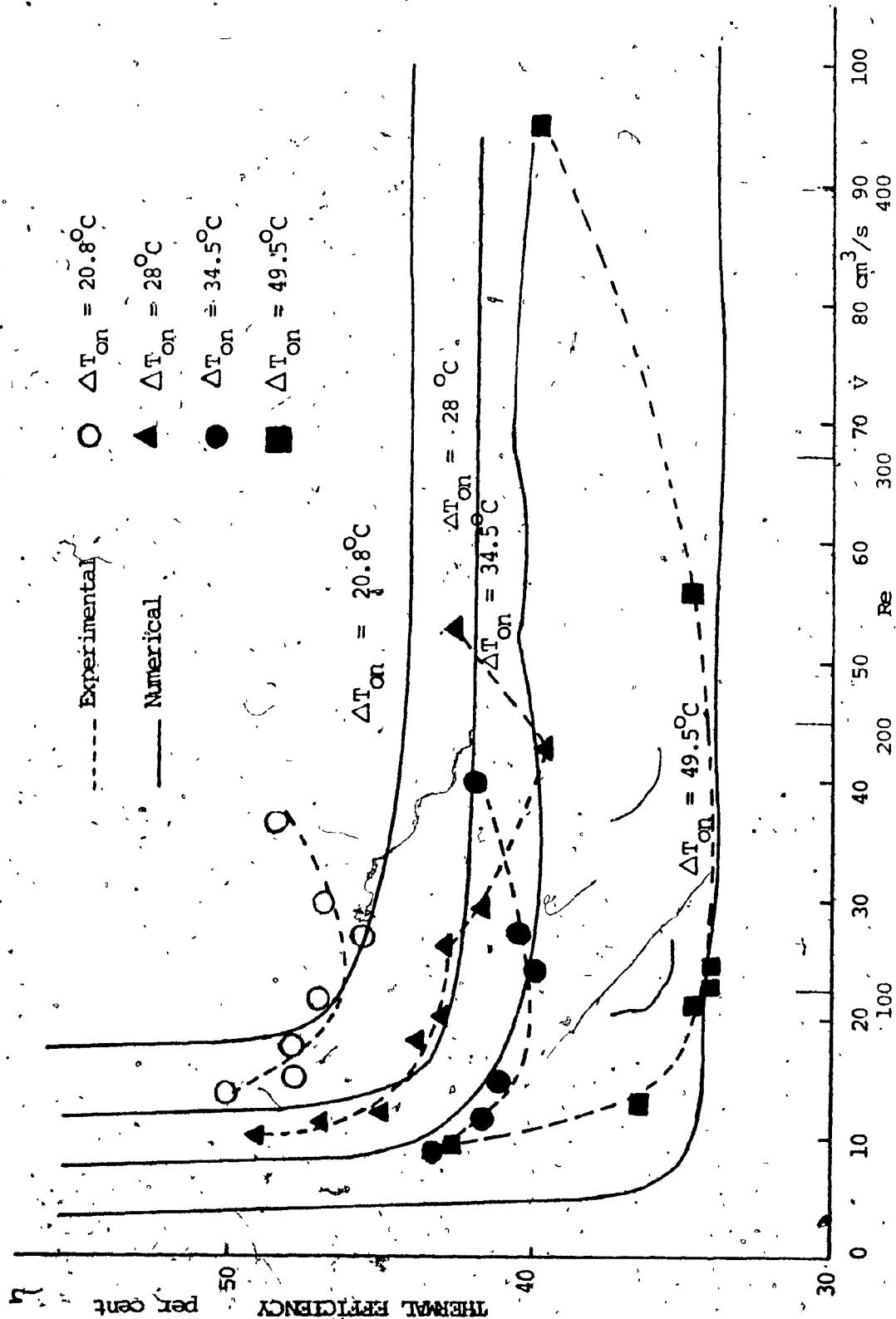


Figure 22. VARIATION OF EFFICIENCY VS VOLUME FLOW RATE & REYNOLDS NUMBER

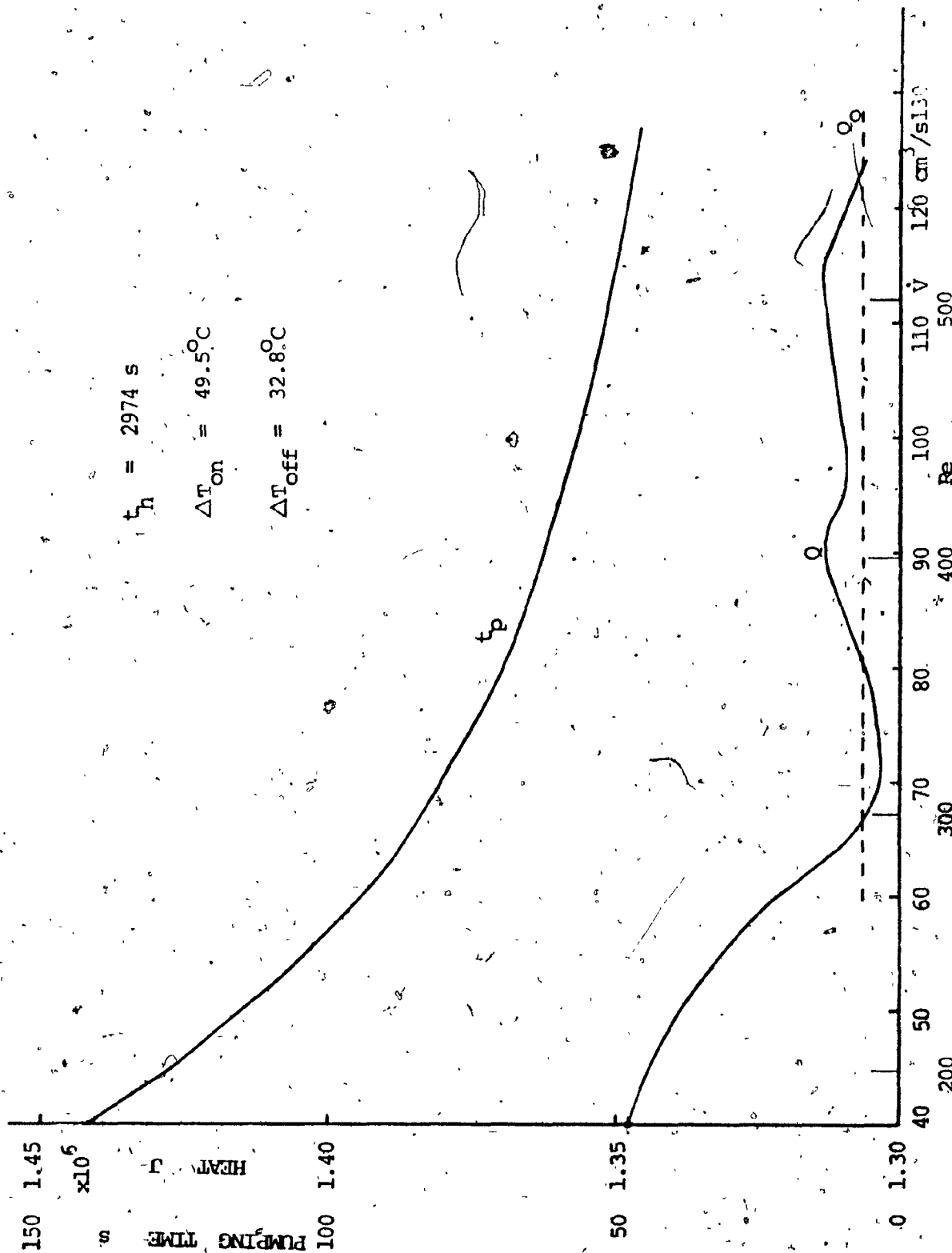


Figure 23. VARIATION OF ABSORBED HEAT PER PERIOD AND PUMPING TIME VS VOLUME FLOW RATE & Re

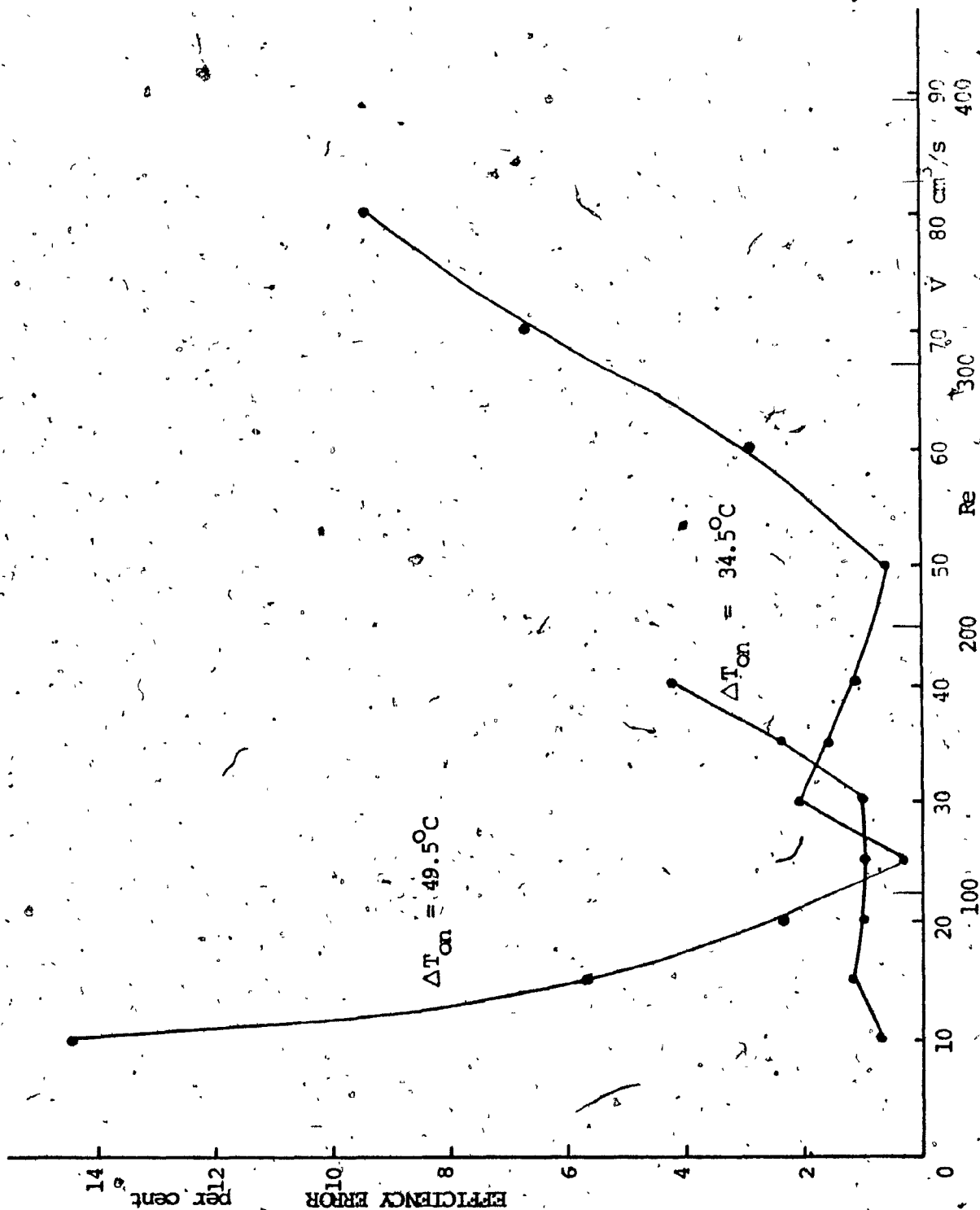


Figure 24. VARIATION OF EFFICIENCY ERROR VS VOLUME FLOW RATE & REYNOLDS NUMBER

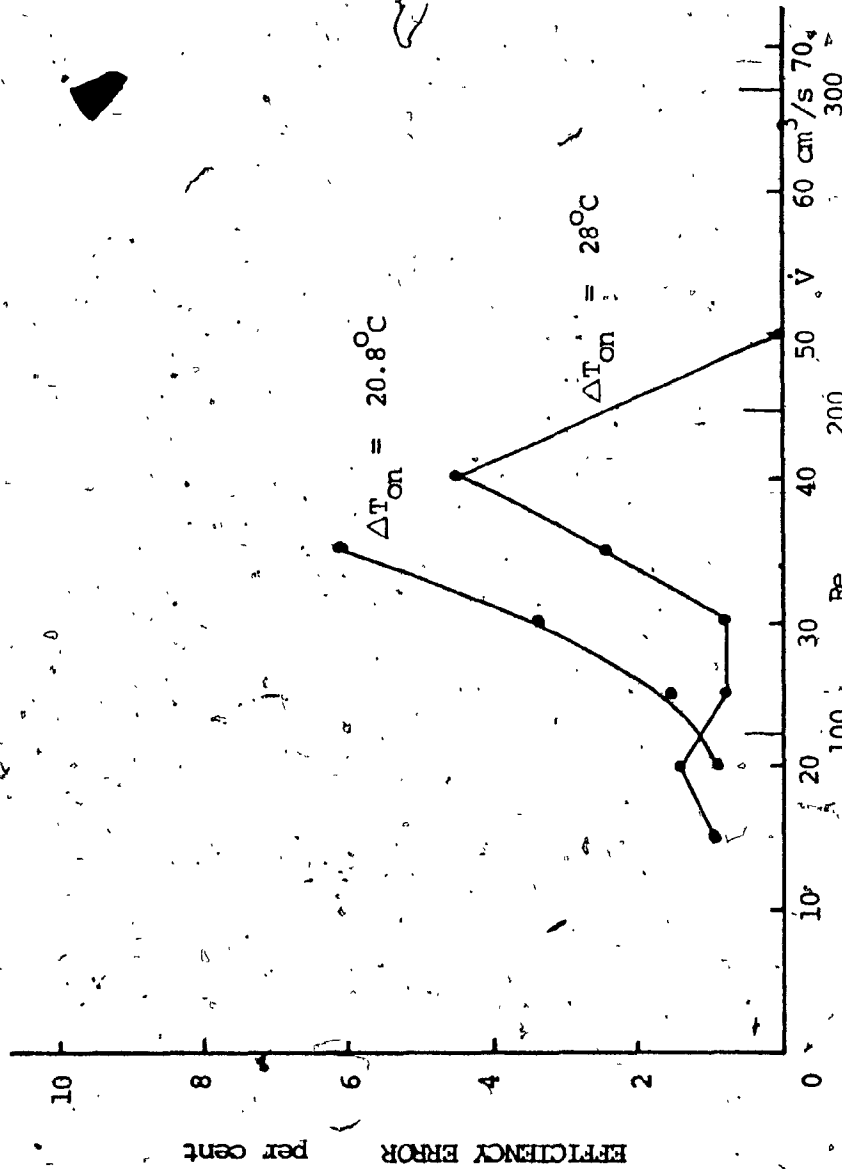


Figure 25. VARIATION OF EFFICIENCY ERROR VS VOLUME FLOW RATE & REYNOLDS NUMBER

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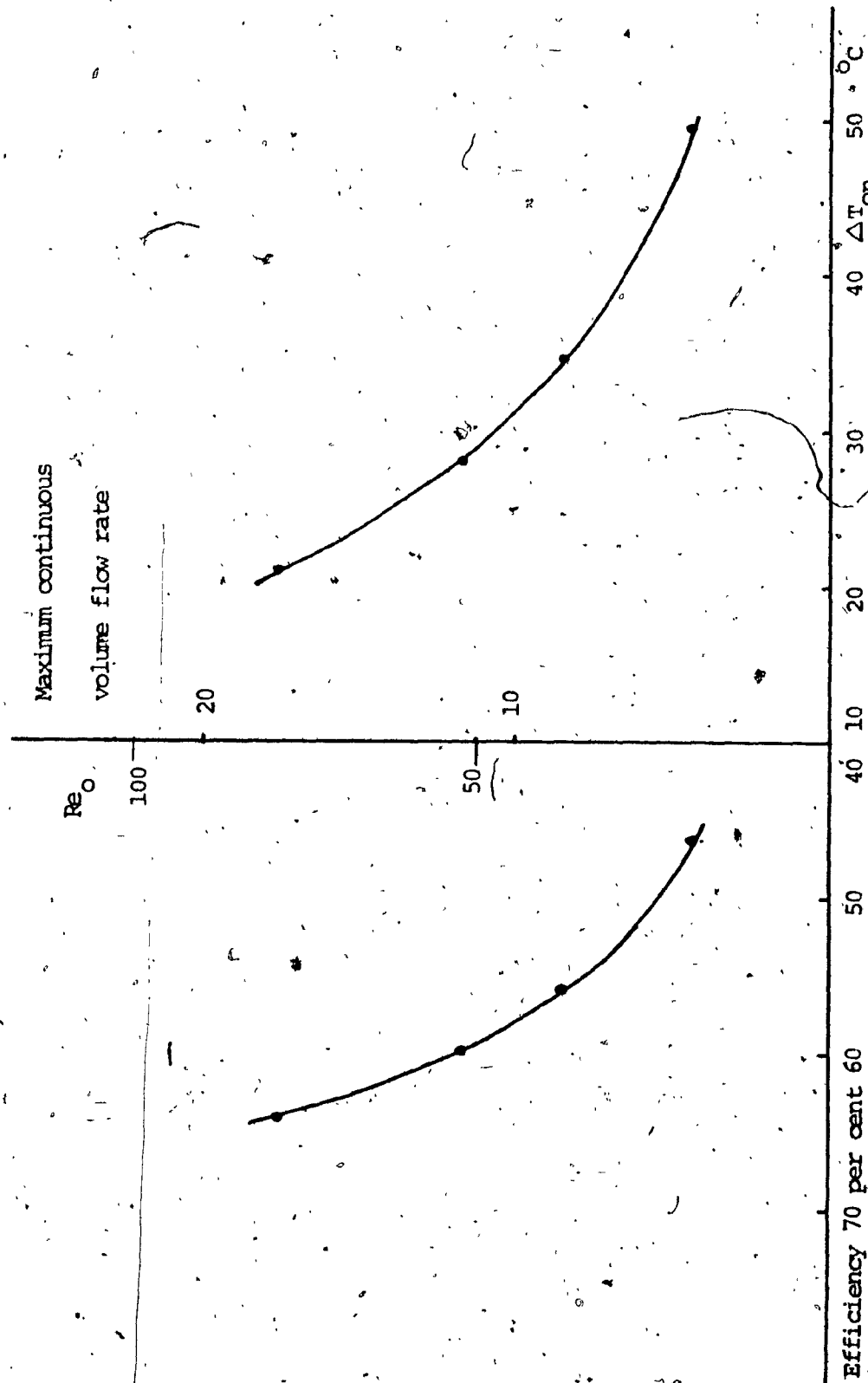


Figure 26. VARIATION OF MAXIMUM EFFICIENCY AND SETTING TEMPERATURE VS MAX. CONTINUOUS FLOW RATE &  $Re_o$